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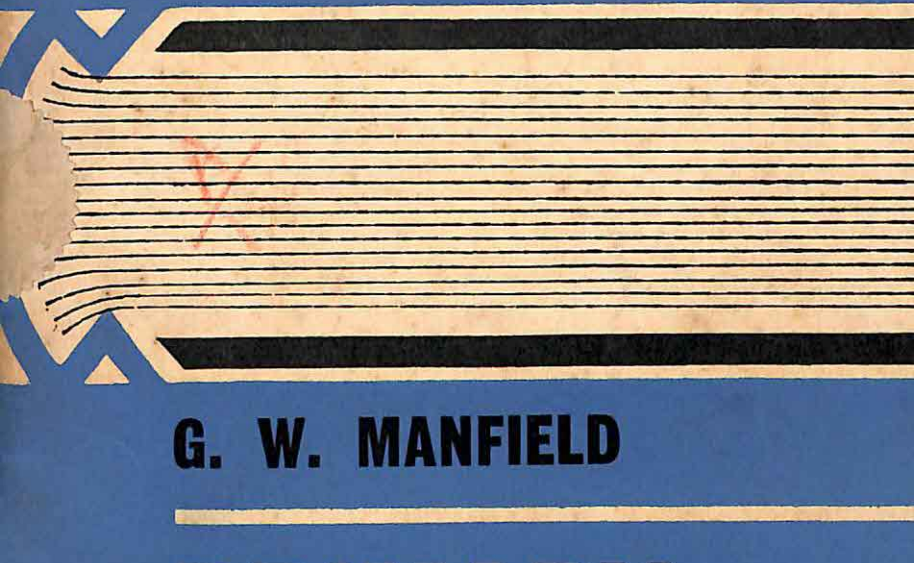

G. W. MANFIELD

# PRIMARY SCHOOL ARITHMETIC

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# Primary School Arithmetic

BY

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## §1. INTRODUCTORY

*"Failure in arithmetic all too often means failure in teaching, not in learning."*

Catherine Stern in
*Children Discover Arithmetic.*

This quotation is worthy of careful attention by all teachers who are responsible for teaching arithmetic. In the case of those who are apt to lay the blame for failure on the child, labelling him as dull or backward or as one who just cannot do arithmetic or is not interested in it, the quotation is worthy of very special thought and consideration. So many teachers regard Arithmetic as one of the subjects they are forced to teach and think of it as a series of mechanical processes which can be drilled into the child by a constant stream of exercises based on a set of precise rules which have been learned by heart, parrot fashion. Little time is spent on planning and little thought is given to method. One exercise follows another through the text book with the natural result that Arithmetic becomes drudgery, pupils lose interest and often develop an active dislike of the subject.

Fortunately, certain general principles can be given which will guide the teacher in his approach to his teaching. If he observes these principles it will certainly help his pupils to learn, and it will go a long way towards developing in them an interest and a desire to learn. Arithmetic is often considered to be a dull subject and even more often a very difficult subject. It need be neither provided the teacher is prepared to give careful thought to the subject itself and to the methods to be used in teaching it.

## Four fundamental principles

All teachers are concerned with the success of their teaching and their success depends upon the success the pupil has in learning what has been taught. Research has shown that for the pupil to succeed in Arithmetic the teaching must be based on four main precepts:

i That the work in Arithmetic must be closely related to the pupil's own experience.
ii That the teacher must have a clear understanding of all that he is aiming to teach. In other words that the teacher understands arithmetic and the reasons for teaching it.
iii That the teaching must be carefully planned to ensure steady but definite progress with ample opportunity for practice.
iv That the teacher must realise that effective and successful teaching depends upon the degree of understanding he has of how the child learns and why he fails to learn.

These are four fundamental principles of teaching which apply, not only to Arithmetic, but to all other subjects. With a difficult subject they are perhaps even more important and the teacher should note them very carefully.

## Extension of the principles

Some more specific details may be of help to the teacher in order to create a background for the work which is described in the following chapters. These extend and expand the four main principles given above:

i *The teacher must preserve the correct balance between the understanding or concept of number and the development of skills or techniques.*
Both are necessary, but technique has little value without understanding. For example, techniques or skills may be taught by rote or by learning by heart. The pupil may be able to repeat that $2+4=6$ without being able to use this technique in a simple problem such as: "John has four bananas. If Sam gives him two then they will each have the same number. How many bananas did Sam have at the start?" The ability to give the answer to the question $2+4$ is of little real value unless it is accompanied by the ability to apply the technique involved to the solving of simple problems.

ii *There must be no haste especially in the early stages.* Moreover, the development of the work must be so planned that new skills are introduced gradually. For example, when teaching addition, the work must be planned so that one difficulty is overcome at a time:

*a* Addition using numbers up to 9—the basic addition facts.

*b* Addition using numbers of two figures, tens and units, without bridging.

*c* Simple bridging with units only.
*d* Addition involving noughts.

leading to the addition of harder numbers (3 figures) without bridging, then with one-figure bridging and later with two-figure bridging and so on.

iii *Fundamental knowledge must become automatic.* Thus, the simple addition and subtraction bonds, the multiplication tables, the tables of the measures must be known by heart, but only when the meaning of them is understood by the pupil. These fundamentals are the "tools" of arithmetic and no craftsman can work without the correct tools. To know the basic facts by heart without hesitation will save a great deal of time and so speed up the work, thus releasing time for other more interesting and valuable topics.

iv *Children learn arithmetic by doing arithmetic.* They do not learn by watching the teacher do arithmetic or by listening to explanations. Arithmetic is the one subject which has to be done by the individual. There must, however, be understanding before the doing of arithmetic is possible.

v *Help is necessary when the pupil experiences difficulty*, but the teacher must know what the difficulty is before he can help. This implies that the teacher must study the errors made. He must be able to distinguish those errors caused by misunderstanding and those caused by incorrect calculation. He must also know the best way to deal with them: he must know when to teach his lesson over again and when to revise basic principles.

vi *Avoid long meaningless calculations which have no real use* and which are not likely to be met in everyday life. Teachers often set long calculations such as: "Reduce 3 tons 5 cwt. 17 lb. to ounces" or "Multiply 8,769,236 by 847,631" or $6\frac{5}{9}\times 4\frac{17}{24}\times 16\frac{37}{48}$. They are quite valueless, they waste a great deal of valuable time, and it is unlikely that pupils will ever meet them when they have left school. For example it has been estimated that over 90 per cent. of the use of vulgar fractions by adults is confined to the simple fractions, halves, quarters, thirds and eighths.

These very long and complicated calculations can do much harm because of the sheer drudgery they involve and children can soon lose interest and any incentive they may have for work. Practice in calculation and the testing of proficiency in the techniques can be accomplished equally well using much more simple questions involving much smaller numbers. The time taken by these long and complicated calculations can be spent with much greater advantage on practical work on say measurement, on simple geometry or on solving problems.

vii *Take care that your blackboard work provides the pupils with a good clear and neat model on which they can base their own standards.*
An untidy and careless blackboard will result in untidy exercise books and careless figures, and consequently there will be mistakes.

What you write on the blackboard will remain on view to the class for a considerable time. If it is to be of the greatest value then it must create the right standards and leave the pupil with the correct ideas. Calculations not needed in the body of the sum should be written on one side just as carefully and neatly. Untidy or scribbled marginal work is the cause of many errors. For example:

| 34 | *Margin* |
|---|---|
| 19)649 | 19 × 1 = 19 |
| 57 | 19 × 2 = 38 |
| 19) 79 | 19 × 3 = 57 |
| 76 | 19 × 4 = 76 |
| 3 rem. | |
| Ans. 34 rem. 3 | |

It should be the aim of all teachers to keep marginal work to an absolute minimum. Much of the marginal work seen in exercise books is unnecessary and results from a lack of training in quick mental work. In order to remove the necessity for marginal work every encouragement must be given to quick mental calculation by regular daily practice. This will train pupils to 'work in their heads' rather than to rely on written calculation on paper. Again much of what is found in the margins will be unnecessary if pupils have an automatic knowledge of the basic facts of arithmetic, the number bonds and the multiplication tables.

In the early stages exercise books divided into squares are most useful, as pupils are helped in arranging their sums and in making figures of large size. Later the squares can give way to lines and finally to plain paper, but if orderly arrangement and accurate working are desired then help and training must be given from the start.

These are all general principles and in the following sections they are illustrated and expanded by actual advice on lesson planning and teaching methods. Merely knowing the principles without the ability to put them into practice is of little value.

In a small book it is impossible to deal with every aspect of the teaching of arithmetic. Emphasis has been placed on the fundamentals and the early years, as these are vital. A number of pages have also been given to Spatial Work or Geometry because of its value and because it is essentially practical. Certain sections of Arithmetic have been omitted altogether mainly because they could not be treated thoroughly in the small space available. However, there are many good books on the teaching of Arithmetic and the teacher should refer to these to expand the material given here.

# PART 1: ARITHMETIC

## §2. THE EARLY STAGES

> "*Arithmetic may be an exhilarating exercise or it may become a nightmare.*"
>
> Ballard in
> *Teaching the Essentials of Arithmetic*

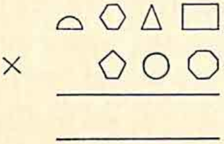
### The necessity for a good start

Obviously the enthusiastic teacher will desire to make Arithmetic what Ballard calls an "exhilarating exercise". By so doing he will create interest and the desire to learn and at the same time prevent or destroy that fear of arithmetic which so many children have.

*Arithmetic is a difficult subject, more difficult for some than for others and therefore the greatest thought and care must be given to teaching it.* So much depends on a correct start to the teaching, and poor teaching methods during the child's first years in school can prejudice the pupil's ability and interest for the rest of his school life.

If you cannot understand where this difficulty lies or how it comes about, set yourself a simple exercise. Imagine that you travel to a new strange world in space where numbers are different from ours. For example:

○ is 1; ⌓ is 2; △ is 3; □ is 4 and so on.
Now work out this sum without using our own numbers but using only the new ones:



This is a much easier problem than that which the young child faces when he first meets our number system and he does not have the advantage of being able to translate the number system into something he does know and understand, as you can. The example given above is an easy one and if you still have doubts let us make the problem rather more complicated. Here is another imaginary number system:

$$\Psi = 1;\ < = 2;\ \nless = 3;\ \rightarrow = 4;\ \varDelta = 5;$$
$$\phi = 6;\ \theta = 7;\ \omega = 8;\ \rho = 9;\ \beta = 0.$$

Now try these simple sums without referring to the table above:

$$\nless + \theta = \quad ;\ \varDelta\beta - <\theta = \quad ;\ \Psi\nless\rightarrow \times <\omega = \quad ;$$
$$\Psi\rightarrow\rightarrow \div \Psi< = \quad .$$

You will realise, I hope, that the simple sums given above are now very much more difficult because you neither know nor understand this new number system. You have not had practice in using it and you should be able to imagine the confusion for the child in the first year of the primary school, when he is presented with a number system with processes of addition and subtraction, multiplication and division, at so fast a pace that there is no time for understanding.

This problem of the beginnings of number teaching is further complicated by the fact that the child is often taught in a language which is not his own. Even if this does not happen at the beginning, it is imposed on him later when the medium of instruction is changed to English.

## The need for understanding

*Children learn best what they can understand* and one of the reasons why you found difficulty with the exercise given above was that you did not understand the new symbols or what they mean. You could easily learn them by heart, but this would not enable you to use them more successfully. This is the first major difficulty you have to overcome—that of understanding. Your teaching must be based on the child's understanding of the abstractions of number and this understanding can only come when the child is given ample opportunity for gaining his own personal experience by dealing with concrete objects—counting them, classifying them, arranging them into patterns, joining them together, grouping them and the like. Even these experiences must be planned, for children are only ready to learn new facts and processes at intervals when they have acquired sufficient experience and understanding and when they have developed the mental ability to proceed further.

## The pre-school period

Before starting school the child has acquired certain ideas of number mainly from his contact with adults and with other children, but also from seeing and handling things in his own simple play. Here then is our past experience and on this we have to base our early teaching. The informal activities of the pre-school years must be continued and developed. Experience must be enlarged and extended.

## The two aspects of Arithmetic

The teaching of arithmetic can be broadly divided into two main parts:

*a The concept of number*—or the understanding of what numbers are and how our number system is constructed.

*b Techniques*—or the actual use of numbers in calculation. The ability to work out the answers to the sums: $19 \times 8$; $17+14$; $31-22$ depends upon the ability to use the techniques of Arithmetic. Many of the results can be learned by heart, parrot-wise. The child can learn that $8+9=17$ without understanding what 8 means or what $8+9$ really means.

Both aspects are important and neither can be neglected. Without an understanding of the number structure the child cannot use numbers intelligently in calculations. Without the technique he will get the wrong answer even if he does understand. *Preserving the balance between these two aspects is one of the teacher's main responsibilities.*

## Extending the child's experience

During the early years in school the emphasis must be on the concept of number. The child must be given the opportunity to understand the structure of our number system and this can best be done through the continuance of his pre-school experience by organised activity in the classroom. Much of this early work must be informal so that the child is given varied opportunity for handling and sorting objects and then in counting them, using, at this stage, the names only. While some apparatus is necessary a great deal can be done with simple pieces of stick, with beans or seeds or beads or with the tops from mineral water bottles. Here are some simple activities which need little prepared apparatus:

i Counting strides or hops using the numbers 1 to 10 orally.
ii Counting sticks into groups of 1 stick, 2 sticks, 3 sticks, etc.
iii Throwing balls into a basket and counting those which go in and those which do not.
iv Stringing beads to make a necklace and counting.
v Counting the parts of the body—one hand, two eyes, one nose; two ears; five fingers and so on.
vi Simple rhymes and jingles. Here is one in English, but the teacher can build others in the child's own language:

> One, two, three, four, five,
> Once I caught a fish alive,
> Six, seven, eight, nine, ten,
> Then I put them back again.

vii The game of "I Spy"—I spy something with four legs, with two wheels and so on.

viii Counting with flags in a sand tray.

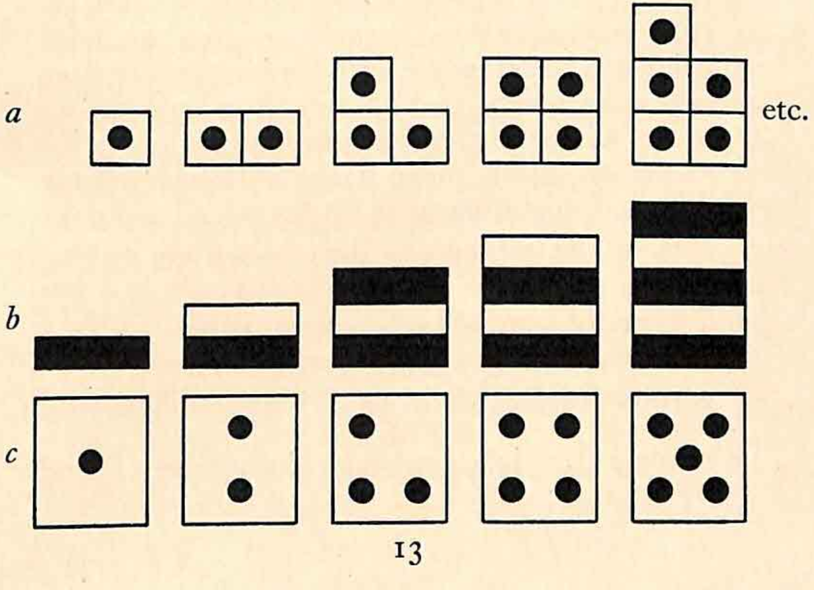
ix Building with bricks and counting.

x Rhythmic counting to the ringing of a bell or the beating of a drum.

There are many more of these simple games which the teacher with initiative can devise for himself.

### From oral numbers to the recognition of symbols

So far no mention has been made of the number symbols and this must be the next stage in developing the concept of number. Here "*Pattern*" *is important as it aids the easy recognition of groups*. There are many "patterns" and three are illustrated, but care should be taken to provide experience of more than one at some stage in the teaching.



Opportunity should be given for the "*pattern number*" to be associated with the same number of actual objects and to link the pattern with the spoken name and later the written symbol. Pattern is useful also because it helps recognition and that shown under (*a*) is particularly useful because it is both simple and distinctive. The odd numbers have odd shapes and the even numbers, even shapes. Later they may be joined together for addition and used also in subtraction and multiplication. For division it is simple to break down a number into smaller groups, eight can be shown to be equal to four twos and two fours. Remember that it is not wise to restrict pupils to one pattern, however useful it may be.

## The written symbols

It is a large step from the oral number to the written symbol and a very important one. To attempt it without sufficient background experience often results in confusion and therefore the introduction of the written symbols must not be rushed. Introduce only two or three symbols at a time and give plenty of varied experience in using them. Here are some of the activities which can be used:

- i Fitting the symbol to the correct number of objects.
- ii Fitting the symbol to the correct pattern.
- iii Fitting the symbol to pictures of objects.
- iv Fitting the symbol to the number of flags in a sand tray.
- v Showing the right symbol for the number of hand claps or rings of a bell.

vi Fitting pegs in a peg board to match a number symbol.
vii Recognition of a number symbol by the use of flash cards.
viii Making or drawing simple pictures to match a number symbol.
ix Counting groups and finding the right number symbol.
x Finding the right number to fit a given number of objects or a pattern or a jig saw.

Again little apparatus is necessary but a supply of large numbers on cardboard will be useful.

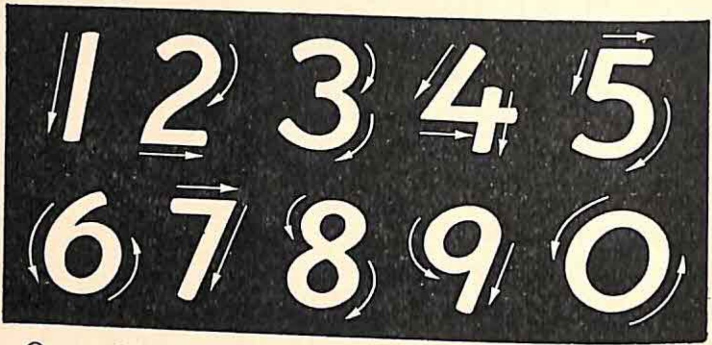
### Writing numbers

Once again patience on the part of the teacher is necessary. Many young children experience great difficulty in writing figures, especially if expected to write them small. Wall blackboards and chalk are probably the best media to use in the first stage but few schools will have these facilities and other methods will have to be used. The child will need all the help you can give him and some of the devices which can be used to provide this help are described:

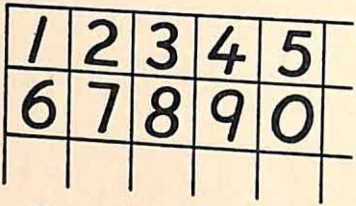
i Sand trays for tracing numbers with the finger.
ii Stencils or cut-out figures which may be used for tracing the numbers.
iii Tracing paper and numbers for tracing through the paper.
iv Dotted numbers for completion, using a coloured pencil.
v Individual free-arm writing boards, made of plywood and painted black for use with chalk.

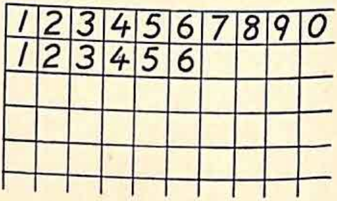
vi Brown paper or cardboard for use with coloured chalk.

Right from the start some guidance should be given on the actual writing or drawing of the number—where to start and where to finish. Here is one method which has been proved to be successful:



Once this stage of large numbers has been completed and pupils have obtained a reasonable proficiency, proceed to smaller numbers using squared paper. The size of the squares can be diminished gradually.





Here then in this chapter are the first steps in teaching. Because of the importance of a good start this has been treated in some detail but there is still much which could be added if space permitted.

The important points for the teacher to remember are:

i Children will learn arithmetic only if they understand the structure of the number system.
ii The pace of the teaching must be slow and must be geared to the child's ability to understand.
iii The aim must be to give the child ample experience by varied activity work in which he sees the need for numbers.
iv "Pattern" is important because it aids recognition and helps understanding.
v Number symbols must be associated with the actual number of objects, the spoken number and the symbol.
vi The writing of numbers is difficult and the ability to write good, clear numbers comes only after practice and guidance.

## §3. THE TECHNIQUES OF ARITHMETIC

### ADDITION AND SUBTRACTION

> *"Progress in Arithmetic as a whole is determined by the degree of proficiency attained in knowing the basic facts of addition, subtraction, multiplication and division."*
>
> Lovell and Smith in
> *The Teaching of Arithmetic in Primary Schools*

The skill of any craftsman depends largely upon his ability to use the tools of his craft. He must know exactly how and when to use a particular tool in order to obtain the desired result. His tools will be well cared for, never being permitted to become dull or rusty and the good craftsman will keep his skill alive by regular practice.

The tools of Arithmetic are the four techniques, addition, subtraction, multiplication and division and the good craftsman in arithmetic will know how and when to use them. The importance of these four techniques will be recognised by all teachers, for they are the basis of all progress.

### Proficiency in fundamentals

Allied with each technique there are certain fundamental facts. For addition there are the simple addition bonds using the numbers 1 to 9 or if 0 is considered to be a number, the numbers 0 to 9.

*Success in addition depends first on an automatic knowledge of these simple addition bonds.* There are other essentials such as a knowledge of the number system and of the principle of bridging.

By "bridging" is meant the general process of crossing over from units to tens, tens to hundreds, hundreds to thousands, etc., and vice versa. It occurs in all the techniques and is often known as the principle of "carrying" or "borrowing". Thus in the multiplication sum, 39 × 3, bridging occurs from the units to the tens and from the tens to the hundreds. In the division sum, 495 ÷ 3, bridging in the reverse direction occurs from the hundreds to the tens and from the tens to the units.

Similarly, for subtraction there are the simple subtraction bonds again using the numbers 1 to 9 (or 0 to 9). For multiplication and division there are the multiplication tables which serve both techniques.

Without a sound knowledge of these basic facts there can be little success in calculation, for all calculation depends upon them.

## The technique of addition

With the varied activity work during the first year at school the notion of addition will have been experienced by all pupils during their informal number games. The idea of "joining on" or adding will have occurred to all in such exercises as, 2 beans and 3 more beans make a group of 5 beans. At this stage the child will probably have physically joined the two groups together to make one group and then found the actual answer by counting. There must be many such experiences, but now the aim must be to dispense with the actual counting and to recognise that $2+3$ make 5 without the use of apparatus or fingers.

There are a number of different ways of expressing addition and it is necessary that the child should be familiar with all of them:

*a* There are four eggs in one box and three eggs in another box. If they are all put together in one box, how many will there be?
*b* I have 5sh. and my brother has 4sh. How much have we altogether?
*c* What number is two more than 5?
*d* I have 6 bananas and my father gives me 3 more. How many bananas have I now?
*e* A line 3 inches long is made 2 inches longer. How long is the new line?
*f* Increase 6 by 4 more.

These can be introduced during oral work and can provide the teacher with a good variety of questions. This is part of the language of Arithmetic and the beginnings of simple problem solving. Time spent on this simple oral work without the burden of written sums will justify itself when the more formal addition sums are introduced. Here, too, is the opportunity for introducing the sign for addition, +, which is also known as the "plus" sign.

In both addition and subtraction it must be made clear that only objects of a similar kind can be added or subtracted. Thus it is possible to add 6 pens and 8 pens and obtain an answer of 14 pens, but you cannot add together 6 pens and 4 bananas.

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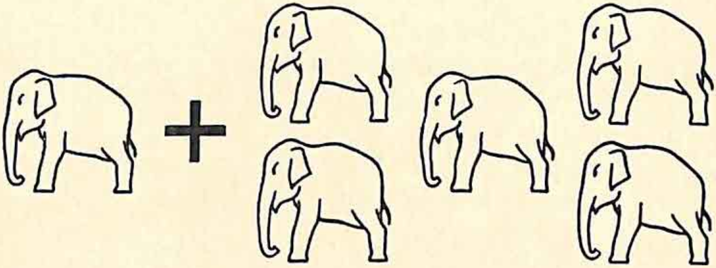
## Introducing formal addition

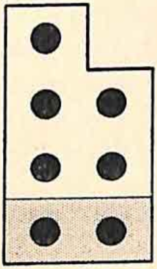
The building of the simple addition bonds using a variety of apparatus can provide the necessary introduction to formal addition. A selection of the more simple apparatus is given, but the enterprising teacher can add to this small selection to provide greater variety and to give further practice:

a *Addition using objects*
b *Picture addition*



**1 + 5 = ☐**

c *Pattern addition*



**5 + 2 = 7**

**2 + 5 = 7**

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## Building the simple addition bonds

The making of simple sums using number cards can be coupled with all the activities used when introducing formal addition. It will be necessary to introduce the "equals" sign, =, and to emphasise that the sign has a number of other meanings such as "make together"; "have together". This can be illustrated by using simple problems such as:

*a* 3 and 5 *make together* 8; 3+5=8
*b* I have 4 shillings and John gives me 5 more. I *have altogether* 9 shillings; 4+5=9
*c* If I increase 7 by 2 more I get 9 altogether; 7+2=9

Pupils should build their own addition bonds by making sums:

$$\boxed{4} + \boxed{1} = \boxed{5} \qquad \boxed{3} + \boxed{6} = \boxed{9}$$

and so learn all about the numbers, that 5 is the same as 4+1 or 3+2, or 1+4, or 2+3 and that 9 is the same as, 1+8, 2+7, 3+6, 4+5, etc.

## Learning the addition bonds

Once children have been given opportunity to build the bonds for themselves and have had frequent practice in using them, the bonds must be learnt by heart. All addition depends upon an automatic knowledge of these bonds and therefore there must be regular and frequent practice in using them. The Addition Table shown will prove a valuable help in these early stages and will help the child to memorise the simple bonds. It can, of course, be extended to include numbers with sums up to 20.

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |
| 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |  |
| 6 | 7 | 8 | 9 | 10 |  |  |  |  |  |
| 7 | 8 | 9 | 10 |  |  |  |  |  |  |
| 8 | 9 | 10 |  |  |  |  |  |  |  |
| 9 | 10 |  |  |  |  |  |  |  |  |

This table allows for addition both ways, i.e. 3+6 and 6+3. When sufficient practice and experience have been given it is advisable to test the results of the work. In all there are 81 combinations for such addition, using the numbers 1 to 9 and 100 if the 0 is included. Tests A, B, C and D contain these 81 combinations using the numbers 1 to 9 only and in Test A only those combinations with answers less than 10 are included. Test B continues these except that the last four questions each have answers of 10. Tests C and D contain the combinations with answers of 10 or over and generally Test D contains the difficult combinations.

| *Test A* | *Test B* | *Test C* | *Test D* |
|---|---|---|---|
| 1+1 | 1+4 | 1+9 | 3+9 |
| 3+2 | 3+1 | 3+7 | 5+6 |
| 2+2 | 4+2 | 5+5 | 6+4 |
| 4+3 | 6+1 | 9+2 | 9+5 |
| 1+7 | 5+3 | 8+2 | 4+9 |
| 5+2 | 4+1 | 7+4 | 6+7 |
| 3+6 | 3+4 | 3+8 | 9+6 |
| 6+2 | 2+1 | 2+9 | 4+8 |
| 7+1 | 1+8 | 4+7 | 8+5 |
| 2+5 | 2+6 | 6+6 | 7+7 |
| 4+4 | 5+1 | 8+3 | 9+9 |
| 2+4 | 2+7 | 9+4 | 8+6 |
| 1+2 | 6+3 | 7+5 | 7+8 |
| 3+5 | 1+5 | 8+4 | 9+7 |
| 5+4 | 2+3 | 5+7 | 8+8 |
| 8+1 | 1+3 | 9+3 | 6+9 |
| 1+6 | 2+8 | 5+8 | 9+8 |
| 3+3 | 7+3 | 7+6 | 7+9 |
| 7+2 | 9+1 | 5+9 | 8+9 |
| 4+5 | 4+6 | 6+5 | 8+7 |
|  |  |  | 6+8 |

There is some difference of opinion on the advisability of introducing the "0" into the number bonds. Some say that such questions as 9+0 have no meaning in so far that you cannot add nothing to nine. However, the nought does occur in all the techniques and the pupil will encounter it when he comes to the more formal work. He will meet such sums as:

$$\begin{array}{r} 140 \\ +\ 64 \\ \hline \end{array} \qquad \begin{array}{r} 286 \\ -\ 40 \\ \hline \end{array} \qquad \begin{array}{r} 104 \\ +215 \\ \hline \end{array} \qquad \begin{array}{r} 203 \\ \times\ \ 6 \\ \hline \end{array} \qquad \underline{4)804}$$

and therefore must realise that $0+4=4$, $6-0=6$, etc.

These Tests are given to illustrate how such tests can be compiled by the teacher himself and he can modify them to include the combinations using the 0. The same type of test will be suggested for use in Subtraction, Multiplication and Division and here the teacher must make his own.

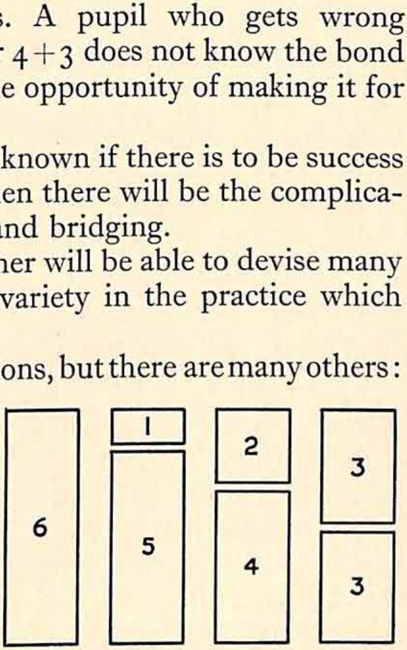
The results obtained from these tests should be studied for weaknesses. A pupil who gets wrong answers for $3+4$ and for $4+3$ does not know the bond and he must be given the opportunity of making it for himself with apparatus.

These bonds must be known if there is to be success with harder addition when there will be the complications of tens and units and bridging.

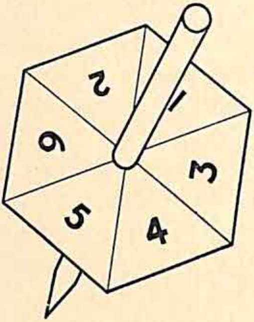
The enterprising teacher will be able to devise many methods for providing variety in the practice which will be necessary.

Here are three suggestions, but there are many others:

i *Cards for finding all the bonds for a given number*



ii *The number top for addition practice*



iii *Addition cards for practice*

| | | |
|---|---|---|
| 1 + 8 | = | |
| 3 + 6 | = | |
| 5 + 2 | = | |
| 4 + 4 | = | |
| 7 + 3 | = | |
| 9 + 1 | = | |
| 6 + 5 | = | |
| 3 + 4 | = | |

| The Story of 7 | |
|---|---|
| 7 = | 1 + 6 |
| 7 = | |
| 7 = | |
| 7 = | |
| 7 = | |
| 7 = | |
| 7 = | |

## Later work in addition

*It is important that progress with the technique of addition should be steady and carefully planned.* In outline the steps in teaching should be based on the following:

- i The simple bonds with answers less than 10.
- ii The bonds with answers over 10 using the numbers 1 to 9 or 0 to 9.
- iii Revision and testing of the bonds.
- iv Addition sums using one and then two figures without bridging.

v Simple one-figure bridging from units to tens.
vi Addition using two or three figure numbers first without and then with one-figure bridging.
vii Two-figure bridging from units to tens and tens to hundreds.
viii General extension of bridging to thousands.
ix Practice with noughts.
x General addition.

Such a plan should be used for all techniques. It will avoid the confusion resulting from mixed exercises chosen without regard for difficulty and it will allow time for each new process to be understood before the next step is introduced.

## The technique of subtraction

As with addition, pupils will have experienced subtraction in their early activity work and all that has been written about early work in addition can be applied equally well to early work in subtraction. Later, when bridging is taught, subtraction becomes more difficult and of course the teacher is then faced with a choice of method.

## Two aspects of subtraction

There are two distinct aspects of subtraction which can best be seen by simple examples:

*a* I have 6 cents, my brother has 4 cents. I have 2 cents more than my brother.
*b* I start out with 10 cents and spend 5 cents. I have 5 cents left.

In (*a*) subtraction is used as a method of comparison as it is in the question: "My school team scored 6 goals and my friend's team scored 2 goals. My school team won by 4 goals." The 4 cents is not contained in the 6 cents and the 2 goals are not part of the 6 goals.

In (*b*) the subtraction is really "taking away". The 5 cents are part of the 10 cents and are actually removed from the 10 cents.

It is important that the teacher should be fully aware of these different kinds of subtraction in order to ensure that questions are mixed and not confined to one type. It is not necessary that they should be defined to the pupils. There are other variations that should be noted:

*a* How many more is 9 than 5?
*b* How many less than 9 is 5?
*c* What must you add to 5 to make 9?
*d* What is the difference between 9 and 5?

## Introducing formal subtraction

When dealing with simple addition three activity methods were suggested as examples. These same methods can be used equally well for subtraction and here are three more which can also be adapted for addition:

i *The story of a number using subtraction*
Using the number 7 the story would be:
If 1 is taken from 7 there will be 6 left,

$$7-1=6$$

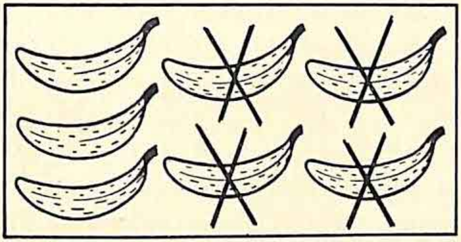
If 2 is taken from 7 there will be 5 left,

$$7-2=5$$

and so on.

ii *Subtraction rhymes* such as the "Ten green bottles", which can be modified and adapted to suit local conditions.

iii *Finding the right subtraction sum card to match a picture*



$$7 - 4 = 3$$

or

$$\begin{array}{r} 7 \\ -\,4 \\ \hline 3 \\ \hline \end{array}$$

Pupils should build the subtraction bonds for themselves using number cards and when the bonds have been constructed they must be learned by heart.

All that has been said on pp. 21-26 applies equally to the early work on subtraction, and similar tests can be made by the teacher for testing the proficiency of his pupils.

### Later work in subtraction

Provided there is no bridging, then subtraction is as easy as addition but because of the increased complication with bridging more care is needed when teaching it. There has been considerable discussion over the methods for subtraction. Some teachers say that the actual method does not matter provided the same method is taught right throughout the school and there is a great deal of truth in this. Others support the use of one of the two methods and can produce convincing arguments to support their views.

The two standard methods of subtraction are:

(*a*) The method of *Decomposition* (or the method of "breaking down").

(*b*) The method of *Equal Addition*.

Whichever of these two methods is used in your school, you will certainly have to start with *Decomposition*; when you first begin to teach subtraction with sticks, or small stones, you will "break down" the bundles or groups of ten to show how the process of bridging works.

With either method, the process must first be taught with sums in which *only one bridge* has to be crossed, from the units to the tens (e.g. 63 − 49). When your class fully understand this, the second step will be sums in which only the bridge from tens to hundreds has to be crossed (e.g. 847 − 362). The third step is crossing both bridges in the same sum (e.g. 643 − 459), and the last step is sums in which the first bridge makes it necessary to cross the second (e.g. 653 − 459).

The examples given here are from Step 3.

## Decomposition

$$\begin{array}{rrr} (5) & (3) & \\ 6^1 & 4^1 & 3 \\ -4 & 5 & 9 \\ \hline 1 & 8 & 4 \\ \hline \end{array}$$

9 units cannot be taken from 3 units, so we take a ten from the top line, i.e. from the 4 tens, so that 3 tens are left. The ten we have borrowed together with the 3 units makes 13 units: 9 from 13 units leaves *4 units*.

There are now 3 tens left in the top line. 5 tens cannot be taken from 3 tens, so we take a hundred from the 6 hundreds, so that 5 hundreds are left. The 10 tens (100) we have borrowed together with the 3 tens make 13 tens: 5 tens from 13 tens leaves *8 tens.*

There are now 5 hundreds left in the top line. 4 hundreds from 5 hundreds leaves *1 hundred.*

*Answer 184.*

## Equal Addition (1)

$$\begin{array}{rrr} 6 & {}^{1}4 & {}^{1}3 \\ -4^{1} & 5^{1} & 9 \\ \hline 1 & 8 & 4 \end{array}$$

9 units cannot be taken from 3 units, so we add a ten (10 units) to the 3 units, making 13 units. 9 units from 13 units leaves *4 units.*

"To be fair" we now add a ten to the bottom line, so that we have 6 tens instead of 5 tens. But 6 tens cannot be taken from 4 tens, so we add 10 tens (100) to the top line, making 14 tens. 6 tens from 14 tens leaves *8 tens.*

"To be fair" we now add a hundred to the bottom line, making 5 hundreds. 5 hundreds from 6 hundreds leaves *1 hundred.*

*Answer 184.*

## Equal Addition (2)

Many teachers use a modified version of the ordinary method of equal addition, as follows:

$$\begin{array}{rrr} 6 & {}^{1}4 & {}^{1}3 \\ -4 & 5 & 9 \\ \hline 1 & 8 & 4 \\ \hline & (1) & (1) \end{array}$$

9 units cannot be taken from 3 units, so we add a ten (10 units) to the 3 units, making 13 units. 9 units from 13 units leaves *4 units.*

5 tens cannot be taken from 4 tens, so we add 10 tens (100) to the 4 tens, making 14 tens. 5 tens from 14 tens leaves 9 tens, but we must now take away the ten we added on in Step 1, and this leaves us with *8 tens.*

4 hundreds from 6 hundreds leaves 2 hundreds, but we must now take away the hundred we added on in Step 2, and this leaves us with *1 hundred.*

*Answer 184.*

Examine these three methods carefully, and make quite sure you understand the differences between them. It does not very much matter which you choose, but it is important that you yourself should always use the same method, and it is very much better if the same method is used throughout the school.

*Crutches:* At first your pupils will need some help to remember what they have done. These helps (or "crutches") are the small figures and figures in brackets in the examples given above. But as soon as possible they must learn to do without "crutches" either in the sum itself or in the margin.

## §4. THE TECHNIQUES OF ARITHMETIC

### MULTIPLICATION AND DIVISION

> *"It is essential that the fundamental processes of arithmetic shall become automatic before the child leaves the Primary School."*
>
> *The Primary School*

### The technique of multiplication

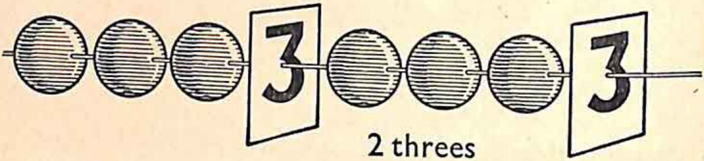
From experiments it has been estimated that multiplication causes about three times as much difficulty as addition and therefore great care and patience must be exercised in teaching it. It has also been estimated that more mistakes in calculation occur through lack of automatic accuracy in the common multiplication tables than from any other cause. As multiplication is probably the technique in greatest use in everyday life the introductory work upon which understanding is built must be very thoroughly done.

In its early stages multiplication is based on addition and should therefore be linked with it and may be introduced once addition up to 20 is firm.

### Introductory work on grouping

Before any mention is made of multiplication, pupils should be given opportunity for a great deal of varied practice in grouping and in the recognition of groups of the same number. This experience of groups is the basis of both multiplication and division and time spent on such introductory work is not wasted. There are many exercises which can be devised to give this experience:

i Threading beads or bottle tops on a wire with a card to indicate the number of objects in the groups.



2 threes

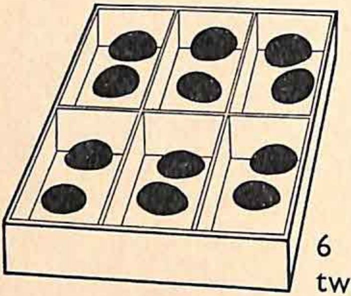
ii Groups of flags on sticks arranged in a sand tray with number cards.

iii Arranging seeds or beans in a box in groups with the number in the group placed with the beans.

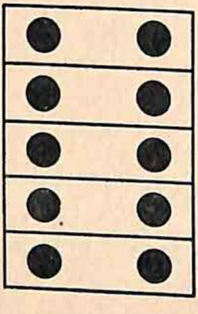
iv Spotting groups of four and naming them: 4 legs on a chair; 4 legs on a table; 4 wheels on a motor car; 4 legs on a cow and so on.

v Arranging cubes on plates for a doll's tea party with the same number on each plate.

vi Arranging pattern cards in groups with the same number in each group.



6 twos



5 twos

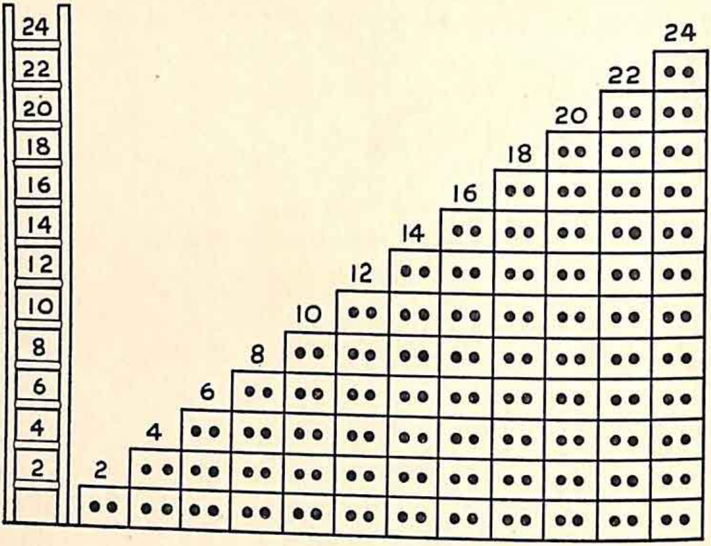
There are many others and the teacher should devise as many as he can. At this stage the important question is: "How many groups of three are there?" and not "How many objects are there altogether?" This is the next stage and should follow when the teacher feels sure that the notion of equal groups is understood and when he is sure that sufficient practice and experience have been given.

## Counting in groups

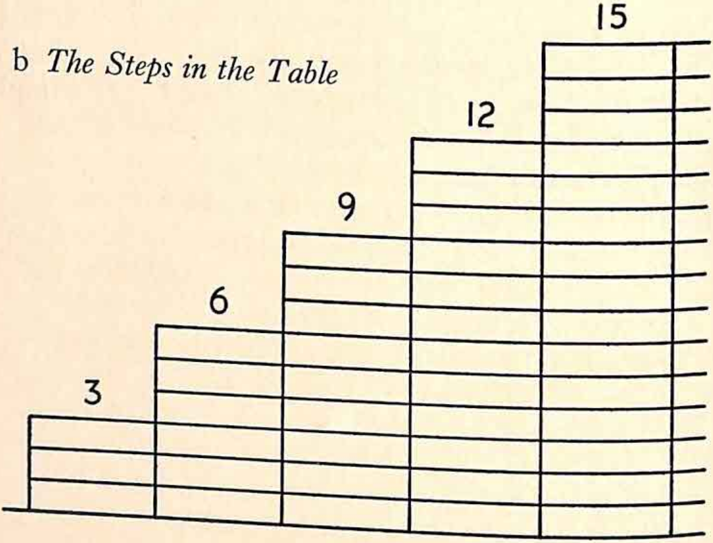
Often the transition from arranging objects in equal groups to counting in groups suffers from incorrect application, pupils being allowed to count in ones. Group counting is essential if there is to be a real and true approach to multiplication. One three—three; another three—six; another three—nine and so on is what should be encouraged and this is, in actual fact, the first step in the building of the multiplication tables. It can be expanded by the use of simple devices:

a *The Ladder game*

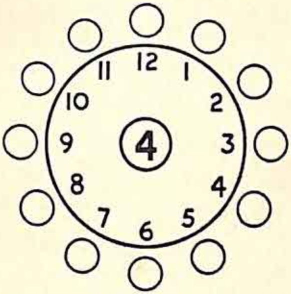
The Ladder of twos, etc. This can be first constructed using pattern cards and then practice can be given in its use.



b *The Steps in the Table*



c Later the Clock Game can be used as a variation and then the simple tables can be compiled.



It is important for the teacher to remember that the simple statement $3 \times 4 = 12$ has two interpretations or translations—three groups of 4 or four groups of 3.

## Grouping of the tables

In the past children were presented with the multiplication tables and told to learn them by heart. They were purely mechanical facts and little if any time was devoted to understanding them or using them intelligently. Today children build their own tables as a result of their own activity; they understand the meaning of them, knowing that they have been built up from their own study of groups.

*Some logical order of presentation of the tables will be advisable and those tables which are connected should be taught together or linked together when they are taught.* Thus the tables of twos, fours and eights could form one set. The tables of fives and tens could form another and the tables of threes, sixes, nines and twelves make another group. Lastly, the tables of sevens and elevens, which do not fall into any group, should be taught.

Even though it is not advisable to teach, say, the tables of threes, sixes, nines and twelves at the same time but to start with the table of twos and the table of threes as the easiest of all the tables, the other tables should be linked with these two basic tables when the time comes to teach them.

## Drill work on the tables

Simply to teach a table and pass on to the next is by no means the end of the task. *The success in multiplication and division depends upon an automatic knowledge of these tables.* This calls for regular practice or drill work and again the aim must be variety. To give the same form of table practice morning after morning will produce boredom. The practice must be stimulating and therefore must be of short duration but at frequent intervals. It should also include practice in the tables in every possible way. Here are some suggestions:

i Clock multiplication—one table at a time with around the class questions using the table both ways, e.g. the value of four twos and how many twos in 16? An enlarged version of the diagram on p. 37 drawn on the blackboard will be necessary.

ii Team questions and answers on tables—the teacher should set the speed of answering.

iii Mixed questions around the class at speed:

(*a*) 5×4 (*b*) How many fours in 24?
(*c*) 36÷4 (*d*) What two numbers when multiplied together give an answer of 12?

iv Numbers arranged in parallel columns. Numbers for multiplying being selected by the teacher by pointing. Immediate answers.

| | |
|---|---|
| 4 | 3 |
| 3 | 2 |
| 7 | 4 |
| 2 | 5 |
| 1 | 6 |
| 8 | 7 |

v Missing numbers from questions written on the blackboard, e.g. $36 \div * = 9$; $4 \times * = 20$; $12 \div 2 = *$ and so on. How many groups of 3 in 27?

vi Giving the stations in the table using the ladder.

## Early work on multiplication

As with all new processes or techniques the teaching must be carefully planned—one step at a time.

i Multiplication using numbers within the tables.

ii Multiplying by one figure without bridging, using multiplicands of two figures only.

Example:

$$\begin{array}{r l} 23 & \text{(multiplicand)} \\ \times\ 3 & \text{(multiplier)} \\ \hline 69 & \text{(product)} \\ \hline \end{array}$$

iii Multiplying by one figure with noughts in the multiplicand—still no bridging.

iv Multiplying by one figure only, with bridging from units to tens only, later from tens to hundreds only.

v Multiplying by one figure only, with bridging from units to tens and tens to hundreds and so on still with one figure multipliers.

Long multiplication will follow when this basic introduction has been completed.

## Basic multiplication facts

As for addition and subtraction, there are basic multiplication facts which the pupil must know or learn. For multiplication up to 12×12 there are 169 such facts, 132 from the eleven tables (2 to 12), 12 from the table of ones (which are the reverse questions of $1\times2$; $1\times3$, etc.) and 25 questions using noughts. There is no difficulty in preparing these questions and it is suggested that they should be arranged on Test Cards with the questions well mixed. Round-the-class questions, competitions and any other device the teacher can find should be used for drill and practice with the test cards to check on progress from time to time.

In addition and subtraction such questions as $2+0$ and $5-0$ are considered by some to lack reality, but they do occur in practice. In multiplication and division such operations as $2\times0$, $4\times0$, $0\div3$ are rather more important, for they often cause difficulties in long multiplication and division.

## Long multiplication

Even with small numbers the method of long multiplication has to be taught as soon as two-figure multipliers are introduced. It is advisable to precede the work by mental practice on multiplying by 10, 20, 30, etc., and to follow this by two-stage multiplication, first without the complication of bridging and later with bridging, e.g.

$$23\times13 \qquad\qquad \begin{array}{rcr} 23\times10 & = & 230 \\ \underline{23\times3\;\;} & = & \underline{\;69} \\ 23\times13 & = & 299 \end{array}$$

This method forms a sound introduction to the more formal method which only differs from it in the way it is set out.

$$\begin{array}{r} 23 \\ \times\ 13 \\ \hline 230 \\ 69 \\ \hline 299 \\ \hline \end{array} \quad \textit{or} \quad \begin{array}{r} 23 \\ \times\ 13 \\ \hline 69 \\ 230 \\ \hline 299 \\ \hline \end{array}$$

Whether the teacher adopts the method of multiplying by the units figure first or the tens figure should be decided by the method adopted by the school. There is less confusion in the minds of pupils if there is a standard method in use in a school. Provided children understand what they are doing then the actual method adopted is of secondary importance. Later they can change from one method to the other without difficulty.

## The use of factors in multiplication

During the early stages of teaching when the mechanics of the technique are being taught it is not advisable to introduce multiplication by factors, as the introduction of another method, even if it results in easier numbers, often causes confusion. Moreover, not all numbers can be split into factors and the method can only be used in special cases. Later, when the technique has been mastered, there is no reason why factors should not be used provided pupils understand what factors are. Often the method provides a short cut as in $24 \times 15$ where it is easier to say $24 \times 5 = 120$ and $120 \times 3 = 360$ than to work a long multiplication sum. While factors must be taught because of their use in finding common denominators and square roots there is no real reason why time should be devoted to teaching multiplication by factors as a special method.

## The technique of division

Division is by far the most difficult of the four techniques. It involves multiplication and subtraction and both are themselves difficult. On the other hand division is probably of least importance in everyday life.

## The two aspects of division

In the early stages of the teaching the two different interpretations are of more concern to the teacher than to the pupil, but later the pupil must understand them if he is to interpret problems correctly.

A simple division sum like 15 ÷ 3 can have two different meanings:

(*a*) *Measurement:* How many pieces of cloth each 3 feet long can I cut from a roll of 15 feet?

*Answer: 5 pieces.*

[How many times does 3 go into 15?]

or (*b*) *Sharing:* Share 15 oranges among 3 boys.

*Answer: 5 oranges each.*

[Divide 15 by 3.]

In the early stages, when understanding is so important, it is necessary that the teacher should be sure that both aspects are covered in the questions and the activity work he plans.

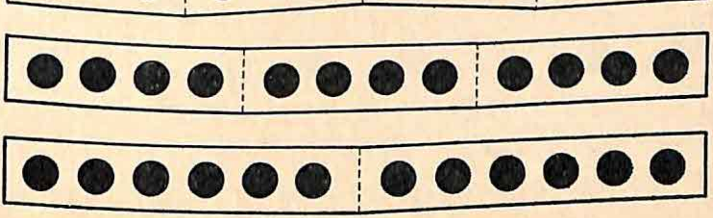
## The approach to formal division

*One of the reasons why pupils stumble over division is that formal division is introduced too early before they have had sufficient background to enable them to understand it.*

In the home the child experiences many examples of sharing when the family has its meals, when wood is collected and shared among those who are to carry it. In his play things are shared and when dancing the dancers arrange themselves into groups. All these and many more are examples of division, although the child does not recognise them as such. When the child attends school, this background must be extended by further examples if he is to approach formal division intelligently and to be proficient and accurate in the technique.

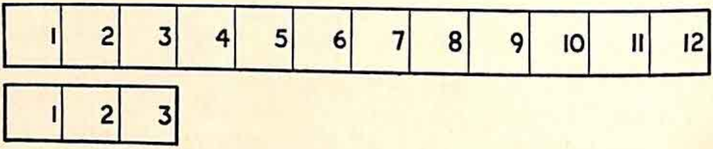
Once again organised activity work will provide the method and a small selection of such exercises is given as a guide for the teacher when preparing the exercises for his own class:

- i Sharing the beans in a bag between two other pupils—later between three, four or five pupils to give the idea of division by the use of groups. This is also useful for early work on remainders.
- ii Flags arranged in groups in a sand tray—useful for groups of 2 flags, 3 flags and so on.
- iii Dividing by folding strips of paper marked with dots.



This method may be used with other numbers and so give early experience in the facts of simple division.

iv Using a paper ruler and a unit strip of (say) 3 inches. Finding how many threes in 12 by moving the strip along the paper ruler.



v Simple rhymes and jingles.

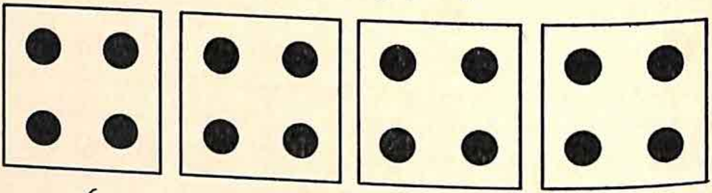
With such exercises must come the words which mean or indicate division: share; divide; split up; how many are there in, and so on. The last named is probably the one which causes most confusion, the reason can be seen from the two questions quoted:

*a* There are 48 oranges in a basket. *How many oranges will there be* in 12 such baskets?—Multiplication.

*b* I have 48 oranges and share them equally among 12 boys. *How many will each boy have*?—Division.

This early work should be accompanied by simple statements to show what has been done.

e.g. Share 16 sweets among 4 boys.



16 sweets, 4 boys—4 sweets for each boy.

This, in turn, will lead to the introduction of the division sign and the simplification of the statement to: $16 \div 4 = 4$.

### Learning the facts of simple division

As for the other techniques there are a number of simple division facts which pupils must learn. These facts are contained in the multiplication tables and provide the teacher with another variation when practising the tables. Thus the tables must be known 4 ways:

$$2 \times 4 = 8; \quad 4 \times 2 = 8; \quad 8 \div 2 = 4; \quad 8 \div 4 = 2$$

In addition, tests should be set using the other methods of writing division sums: $2\overline{)12}$ and $\frac{12}{2}$ and these tests should cover comprehensively all the basic division facts, first with small dividends and later with larger numbers, but all within the tables. Test Cards, some dealing with a mixture of questions and others dealing with the division facts for such numbers as 20, 24, 36 will prove useful.

### Remainders

*There is no reason why remainders should not be introduced into this early activity work.* It is not difficult for the child to understand that 18 eggs shared among 6 people gives an answer of 3 eggs to each person, while if 19 eggs are shared each person will still have 3 eggs but there will be 1 egg left over which cannot be shared. Difficulty and confusion with remainders in later work on division are often caused by lack of understanding of the problem during these preliminary activities.

### Long or short division

This is an old problem and one about which there is still considerable difference of opinion. Many teachers prefer to teach short division first and later to pass on to long division. They argue—and there is considerable strength in their reasoning—that the child will be expected to work such questions as $128 \div 4$ or $\frac{128}{4}$ mentally and here the simple short division habit is more suitable. The child says, 4 into 12 tens goes 3 tens; 4 into 8 units goes 2 units; answer: 32. Many other teachers can see no reason for teaching two methods when the method of long division is universal. Compare the two methods, which differ in form rather than in technique:

$$\begin{array}{rcccc}
7) & 2 & 2 & 5 & 4 \\ \cline{2-5}
 & & 3 & 2 & 2
\end{array}
\qquad\qquad
\begin{array}{rcccc}
 & & 3 & 2 & 2 \\ \cline{2-5}
7) & 2 & 2 & 5 & 4 \\
 & 2 & 1 & & \\ \cline{2-4}
7) & & 1 & 5 & \\
 & & 1 & 4 & \\ \cline{3-5}
7) & & & 1 & 4 \\
 & & & 1 & 4 \\ \cline{4-5}
 & & & . & .
\end{array}$$

The teacher himself must decide, for here there are no overwhelming arguments in favour of either method, but it is as well to remember that methods which are comparatively simple to pupils in the sixth year of the Primary School are not always equally simple to pupils in the second year.

## Long division

Long division has to be taught, difficult though the process may be, for the method of short division can only be used when divisors are not greater than 12. *It is important to proceed slowly and to permit every possible aid in the early stages.* Here is a method of approach which has been found successful:

i Using the table of twos, build up the table of twenties; in the same way build up tables of thirties, forties and fifties, etc., from the tables of threes, fours, fives, etc.

ii Introduce the method of long division using divisors of 20, 30, etc., without crossing off the noughts. Use these simple divisors to give practice in the method and to demonstrate the layout. No remainders.

$$\begin{array}{rl} 4 & \text{(quotient)} \\ \text{(divisor) } 40\overline{)160} & \text{(dividend)} \\ \underline{160} & \\ \dots & \text{(remainder)} \end{array}$$

iii Introduce simple remainders into the sums using divisors as in ii.

iv Prepare a table as below to use with divisors 13 to 19.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 13 | 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 |
| 14 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 |
| 15 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 |
| 16 | 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 |
| 17 | 17 | 34 | 51 | 68 | 85 | 102 | 119 | 136 | 153 |
| 18 | 18 | 36 | 54 | 72 | 90 | 108 | 126 | 144 | 162 |
| 19 | 19 | 38 | 57 | 76 | 95 | 114 | 133 | 152 | 171 |

v Use divisors of 13 to 19 without remainders.

vi Introduce remainders.

vii Extend to other divisors—using a skeleton table to aid division. Allow pupils to write this skeleton table by the side of the sum.

Long division lends itself to long and complicated sums such as: 13942)8649761. These are of little value, being completely unreal, and they are long enough to cause boredom and consequent inaccuracy. Very few, if any, children, or even adults will ever meet them, so why not keep divisors mainly to those numbers which are encountered in everyday life: 36 (inches in a yard); 22 (yards. in a chain); 52 (weeks in a year); 24 (hours in a day); 16 (ounces in a pound); 14 (pounds in a stone). By using such divisors at least your questions can be made to apply to everyday calculations which are useful. Moreover, a great deal of time and energy are released for other more useful and interesting topics.

## The need for practice

Mention has been made of the necessity for practice and for the automatic knowledge of the basic facts. *One of the main causes of errors in division is an imperfect knowledge of the multiplication tables.* However modern the teacher's outlook may be, he must recognise that one of the aims of doing arithmetic is still to get the right answer. In division this can be aided by an automatic knowledge of the multiplication tables and of the subtraction bonds. Drill work on these is therefore essential if progress is to be made and this drill work should go right through the school even if the amount decreases with the years.

## Summary of English phrases used in the techniques

### i *Addition*

- *a* I have 3 bananas, my brother has 2 bananas. *How many are there altogether?*
- *b* *What do 4 and 3 make together?*
- *c* One line is 2 in. long and another is 3 in. long. If these are joined together, *how long will the line be?*
- *d* There are 3 eggs in one basket and 5 in another. *How many will there be* if we put them all into one basket?
- *e* I picked 5 bananas from one tree and 4 from another tree. *How many bananas is this equal to?*
- *f* *What is 6 plus 3?*
- *g* *What is the sum of 4 and 5?*
- *h* *Add 3 to 9.*
- *i* *Increase 6 by 5 more.*

### ii *Subtraction*

- *a* *Take 7 from 16; 16 take away 7; subtract 7 from 16.*
- *b* *What is the difference between 7 and 16?*
- *c* *What is 16 minus 7?*
- *d* *How many less is 9 than 14?*
- *e* *How many more is 14 than 9?*
- *f* *What must I add to 8 to make 13?*
- *g* *How many are left when I take 8 from 15?*
- *h* *How many are left when I take 8 from 13?*
- *i* *How much bigger* (taller, heavier, longer) *is* A *than* B?

iii *Multiplication*
  - *a How many* eggs are there in 4 groups of 3 eggs each?
  - *b What is nine times six?*
  - *c Multiply* 8 by 4.
  - *d What is the product of* 7 and 3?

iv *Division*
  - *a How many* sixes are there in 36?
  - *b How many groups of* 4 are there in 24?
  - *c Share* 8*s*. equally among 4 boys.
  - *d How many times* does 8 go into 64?
  - *e What is the quotient* when 48 is divided by 6?
  - *f Divide* 24 by 3.
  - *g Divide* 6 into 36.

## The importance of good blackboard work

In Arithmetic there is probably more blackboard illustration than in any other school subject and therefore poor and untidy blackboard work will have a very harmful effect. The essential part of all Arithmetic is the figure, but the correct arrangement or layout of sums is also important. Here are the two most important aspects of the teacher's blackboard work in arithmetic:

i *Figures:* they must be clear, correctly formed and must be spaced so that the number is plainly visible to everyone.

ii *Layout:* the lines of figures in a number must be straight and level; figures must be placed correctly in the right columns—units under units, tens under tens and so on.

The appearance of the sum must be orderly and neat with lines carefully drawn. In problem work the writing must be of the same standard as that indicated in the previous section.

Side working must also be as neat as the working in the main sum.

Cancelling lines in fractions must be clear—there must be no doubt which number is cancelled.

If you have difficulty in keeping your blackboard writing straight or your figures of equal size, try drawing guide lines on the blackboard in pencil.

If your blackboard work is good then you can expect a similar standard in the exercise books. Scribbled work is the underlying cause of many errors.

## §5. PROBLEMS AND SHORT METHODS

> *"Arithmetic should not mean to a child only tables learnt by heart and rules of thumb; if it does then intelligent children will be bored, and advanced work made impossible."*
>
> R. K. and M. I. R. Polkinghorne in
> *Fundamental Number Teaching*

To counteract any wrong impressions that might be given by the previous sections on Techniques, here in this chapter the value and importance of problems are discussed. Proficiency in the techniques is very important, but is is not enough by itself. *Equally important is the ability to know when and how to apply these techniques to the solving of problems.* As a general rule the greatest use of Arithmetic in everyday life is in the solving of simple straightforward problems.

The early activity work outlined in very brief form in a previous section is essentially problem work. Yet once this stage is over so many teachers pass to mechanical exercises and neglect problems. Mechanical exercises are of course easier to set, easier to mark and give an impression of efficiency even if the impression is a false one. Mechanical exercises are necessary, but their only really important value is that they provide the pupil with the tools for solving problems.

From the earliest stages the approach to Arithmetic should be through the problem and through a problem which is within the pupil's experience.

## What are problems?

First let us be sure what type of question constitutes a problem. To translate a mechanical question into words, for example, "To thirty-four add twenty-one", does not make this question into a problem. Provided the child can read, can write down the numbers 34 and 21 and knows how to add them, then he can work this sum in words as readily as he can work $\begin{array}{r}34\\+21\\\hline\end{array}$ in figures. However, the question, "I have 21 stamps and my brother has 13 more than I have. How many have we altogether?" does constitute a problem. The pupil is not told exactly what to do. He has to think and to interpret or translate the question into a mechanical technique and he can only be successful when he can correctly interpret the question. *The test of a problem is whether or not the pupil has to think when solving it.*

## Problem solving

While there are definite rules for addition and for all the mechanical processes there are no set rules for problem solving. A method which applies to one problem does not necessarily apply to another. *Problems are almost as individual as the pupils.* The teacher can, however, help his pupils to tackle problems. The first essential is to start problems from the first day in school and to keep at them right from the start. Then the teacher must encourage his pupils to read the question carefully and to interpret its meaning. This is almost the same as translating the question from language to Arithmetic. Take this type of problem:

> "Make the smallest number possible using all the figures 1, 4, 7 and 6. Now make the largest possible number using the same figures."

By questioning, pupils can be led to understand:

i That there are two questions and that two four figured numbers have to be made.
ii That in any number, in this case a number of 4 figures, each figure has a definite place value.
iii The place value of the figure increases by ten times as we move to the left: 4444 = 4000 + 400 + 40 + 4.
iv That for the largest number the figure of greatest value, i.e. the 7, must come first, etc.
v That for the smallest number the figure of smallest value, i.e. the 1, must come first, etc.

There is a very great difficulty when the language of the problem is not the vernacular language of the pupil. Moreover, the vernacular language is often unsuitable for the precise interpretation of the actual arithmetical processes involved. Consequently, pupils will encounter the greatest difficulty in the exact interpretation of a problem unless teachers adopt methods to meet this difficulty. In the early stages it will be advisable for teachers to translate the problem into the vernacular and to introduce the text-book problem gradually when the pupils' understanding of English is more developed. Much of this work should be done in connection with the activity and practical work of the first two years of the Primary school when language and Arithmetic can be taught side by side. It will be necessary, however, to continue the method in decreasing degree throughout the Primary school, a principle which illustrates the saying that "every teacher is a teacher of language".

The making up of problems by the pupils themselves also helps in the understanding of other problems. The expression of those problems from their own experiences or practical work in words helps to show how problems are made up and this helps pupils to interpret the problems they meet in the textbook.

Later, with problems in simple geometry, pupils should be encouraged to make simple sketches to aid the translation.

## Marking problem exercises

Mechanical exercises are easy to mark, for the answer is either right or wrong and the method is clear, for no interpretation is necessary. *With the marking of problems, however, there is the twofold aspect to consider, the method used and the accuracy of the answer.* Both aspects are important, for the one is of little value without the other. In the marking, credit must be given for both method and accuracy. There is no fixed ratio for these marks. A problem which is difficult to interpret and easy to calculate will deserve a different marking from an easy problem with a more difficult or complicated calculation.

*When marking it is important that the pupil should know whether there has been an error in method or in calculation;* where the method, even if accurate, is considered to be poor, or where it can be shortened. To mark a problem wrong without any indication of the nature of the error is no encouragement to the pupil. Discussion of wrong or confused methods should follow where necessary, while the pupil can be set to correct his own errors in calculation.

### Traditional problems

There are many interesting and fascinating problems based on the number system. These problems provide opportunity for discussion on "pattern" and often serve to stimulate interest in the study of numbers later on. Here are three of these traditional problems as an illustration. There are many more.

(i)

$$\begin{aligned} 11 \times 11 &= 121\\ 111 \times 11 &= 1221\\ 1111 \times 11 &= 12221 \end{aligned}$$

Now write down without working the answers for: $11111 \times 11$; $111111 \times 11$, etc.

(ii)

$$\begin{array}{lr} & 86\\ \text{reversed} & -68\\ \hline & 18\\ \text{reversed} & +81\\ \hline & 99 \end{array} \qquad \begin{array}{r} 51\\ -15\\ \hline 36\\ +63\\ \hline 99 \end{array} \qquad \begin{array}{r} 72\\ -27\\ \hline 45\\ +54\\ \hline 99 \end{array}$$

Can you find a reason why the answer to all such sums is 99?

(iii) $1 \times 8 + 1 = 9$
$12 \times 8 + 2 = 98$
$123 \times 8 + 3 = 987$
continue for three more lines.

The value of such problems lies in the interest they arouse and they prove useful for those odd minutes at the end of a lesson when it is not worth while starting on a new exercise or a new topic. The wise teacher will always have something like this ready for these odd moments.

### Short cuts in calculation

No mention has yet been made of the short cuts in calculation which not only save so much time but also develop in pupils an awareness of methods of saving labour. Because of this quality such methods can be considered as problems rather than as mere mechanical calculations. Here are some examples to illustrate some of the methods that should be practised:

i $86 + 99 = 86 + 100 - 1 = 185$
ii $74 + 104 = 74 + 100 + 4 = 178$
iii $382 - 101 = 382 - 100 - 1 = 281$
iv $66 \times 9 = 66 \times 10 - 66 = 660 - 66$
v $17 \times 99$
vi $54 \times 102$
vii $66 \times 98$
viii Multiplying by 10, 100, 1,000, etc.
ix Dividing by 10, 100, 1,000, etc.
x Multiplying and dividing by 25, 50, 125, 250, etc.
xi Such calculations as: $13 \times 12 + 7 \times 12$; $11 \times 17 \times 5 \div 85$

xii Approximate answers to such questions as:
7 yd. 2 ft. $11\frac{1}{2}$ in. × 64; $3{\cdot}98 \times 5{\cdot}01$; $14{\cdot}99 \div 4{\cdot}9$;
sh. $6{\cdot}96 \times 7$; $55\frac{1}{2}$ lb. × 20 in cwt.

There are many others and teachers should watch for them in the questions they set.

## Estimation and approximation

It is most useful if the teacher of Arithmetic encourages and trains his pupils to make some estimate of the approximate answer to a question before they start on it. This will do much to prevent ridiculous or impossible answers and will also enable pupils to realise when an answer is obviously wrong. For example, a class of pupils when set to measure the heights of the boys in the front row of their class and to calculate the average height of these boys, gave answers which varied from an average height of eight inches to an average height of 8 ft. 3 in. This could have been prevented if the class had formed the habit of estimating a rough answer first—even if the estimate were as approximate as, say, between 3 and 4 feet. Even this rough estimate would prevent a ridiculous answer such as 8 inches or 8 ft. 3 in.

Here is a guide to the questions pupils should be trained to ask themselves:

i What will be the units of the answer—will it be a number, a sum of money in shillings and cents; a length in yards, feet and inches or miles, a weight in perhaps pounds and ounces or an area in square inches?

ii Will the answer be large or small? What will be a rough estimate of the answer?
iii Is the answer I have obtained in the correct units (see i above)?
Is it a sensible answer?

Practice can also be given in working out approximate answers and a specimen set of questions is given to show the type of question which can be set. It is not complete, but it will give the teacher some ideas and he can expand them to suit his own needs.

### Rough or approximate answers

Find rough answers for the following as quickly as you can:

| | |
|---|---|
| 1. 4*s.* 11*d.* × 15 | (5*s.* × 15 = 75*s.*) |
| 2. 8*s.* 11*d.* × 11 | (9*s.* × 11 = 99*s.*) |
| 3. 49 × 17 | (50 × 17 = 850) |
| 4. $3\frac{3}{4}$ oz. × 20 in lb. | ($\frac{1}{4}$ lb. × 20 = 5 lb.) |
| 5. 1 gall. 3 qt. $1\frac{1}{2}$ pt. × 63 | (2 gall. × 63 = 126 gall.) |
| 6. 59 × 11 | (60 × 10 = 600) |
| 7. 7 ft. 11 in. ÷ 8 | (8 ft. ÷ 8 = 1 ft.) |
| 8. Find average of 21, 24, 26, 23 | (23—or at least between the largest number (26) and the smallest number (21) |
| 9. 3·9 × 1·9 | (4 × 2 = 8) |
| 10. 15·9 ÷ 3·9 | (16 ÷ 4 = 4) |
| 11. Find total weight of 48 bags each weighing 55 lb. | ($\frac{1}{2}$ cwt. × 48 = 24 cwt.) |

12. $6\frac{3}{4} \times 1\frac{7}{8}$ $(7 \times 2 = 14)$

13. $830 \div 49$ $(830 \div 50 = 16{\cdot}6)$

14. Find average weight: 4 st. 11 lb.; 3 st. 12 lb.; 5 st.; 4 st. 2 lb. (av. of 5, 4, 5, 4,— approx 4 st. 7 lb.)

15. If 12 articles cost 42*s.* what will 23 cost at the same rate? (less than 84*s.*)

## Making sensible answers

In the following statements a decimal point has been left out. Put the decimal point in the right place in order to make the answer sensible.

1. John won the high jump at a height of 45 ft.
2. The train travelled at an average speed of 304 m.p.h.
3. $4{\cdot}5 \times 2{\cdot}1 = 945$
4. $24 \div 0{\cdot}5 = 480$
5. My brother is 575 ft. tall.
6. My brother is 4 ft. 6 in. tall and I am 5 ft. 6 in. tall. Our average height is 50 ft.
7. $1{\cdot}0 \times 0{\cdot}1 = 1$
8. 1 qt. = 25 gall.
9. $14{\cdot}3 \times 2{\cdot}7 = 3861$
10. $6.3 \div 2.1 = 300$

## §6. TESTING AND MARKING IN ARITHMETIC

> "*A question is asked in abstract arithmetic. The answer is right or wrong. There is no compromise; there can be no compromise.*"
>
> Ballard in
> *Teaching the Essentials of Arithmetic*

To some this quotation will appear to be very dogmatic, and there will be a number who will disagree with its implications. To these I would pose a question: "Is there any real value in the working of such an abstract question as 143 × 27 if the answer obtained is wrong?" Whether this wrong answer is due to carelessness or to a lack of knowledge of the multiplication tables or of place values or of the addition bonds is beside the point. The pupil has gained nothing of value from working it incorrectly.

### The need for testing

Almost all Arithmetic is a form of testing. Every sum the pupil works, orally or on paper is a test because Arithmetic is essentially an activity. *You can learn Arithmetic only by doing Arithmetic yourself.* You cannot learn by listening or by watching others do Arithmetic or by reading about it. You have to experience it yourself and so every question is a form of test.

However, in this chapter, Tests and Examinations refer particularly to those exercises set with the definite purpose of discovering something, as distinct from exercises set to give practice in a technique or to give experience in problem solving.

There are two main reasons why tests of this nature are necessary:

i It is important, from time to time, to measure the progress of individual pupils.

ii It is necessary to check on the knowledge of the basic facts of Arithmetic at regular intervals, especially before the teaching of a new topic.

## Compiling tests in Arithmetic

The first essential of any test prepared for use with a class is that *it should have a definite purpose* and the teacher must be quite clear about this purpose. Tests compiled in a haphazard way by the random selection of questions from the text book can accomplish very little except to give the pupils practice.

Next in importance come the actual questions. If these are too difficult then the test will not fulfil its purpose as many pupils will be unable to work the questions and only a few will be able to complete the test. Questions which are too easy will not extend the pupils unless the test is on basic facts when a definite time limit should be imposed. *Choose your questions carefully so that your test can accomplish its purpose and be sure that you know exactly what you hope to learn from each question you set.*

## Specimen tests

Short specimen tests are given as the best method of illustration of the various types of tests and the purposes for which they are set.

(i) *A test on simple addition bonds before teaching "bridging"; pupils are to write down only the answers.*

7+3
9+1
8+4
6+6
9+5
2+8
8+3
9+2
3+9
8+9
4+6
4+9
7+5
6+7
7+7
5+5
5+8
4+7
4+8
8+6

The purpose of this test is twofold, firstly to test the knowledge of the simple addition facts (see §3, p. 24 for full Test) and secondly to prepare for simple addition with bridging from units to tens. A time limit should be imposed—about 2 minutes for the 20 questions.

(ii) *Mechanical Test on Subtraction with one-stage bridging involving noughts.*

| | | | | |
|---|---|---|---|---|
| $\begin{array}{r} 40 \\ -22 \\ \hline \end{array}$ | $\begin{array}{r} 90 \\ -34 \\ \hline \end{array}$ | $\begin{array}{r} 60 \\ -47 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ -33 \\ \hline \end{array}$ | $\begin{array}{r} 50 \\ -38 \\ \hline \end{array}$ |
| $\begin{array}{r} 340 \\ -126 \\ \hline \end{array}$ | $\begin{array}{r} 223 \\ -117 \\ \hline \end{array}$ | $\begin{array}{r} 451 \\ -144 \\ \hline \end{array}$ | $\begin{array}{r} 344 \\ -239 \\ \hline \end{array}$ | $\begin{array}{r} 857 \\ -48 \\ \hline \end{array}$ |
| $\begin{array}{r} 563 \\ -204 \\ \hline \end{array}$ | $\begin{array}{r} 302 \\ -9 \\ \hline \end{array}$ | $\begin{array}{r} 781 \\ -302 \\ \hline \end{array}$ | $\begin{array}{r} 359 \\ -259 \\ \hline \end{array}$ | $\begin{array}{r} 627 \\ -419 \\ \hline \end{array}$ |

(iii) *Mixed Problems on Time*

How many minutes is it to 5·0 o'clock at 4·35?
How long is it from 10.30 a.m. to 11.15 a.m.?
Change 1 hr. 25 min. to minutes.

How many hours are there from noon to midnight?
How many minutes are there in ¾ of an hour?
How many days altogether in January, February and March 1961?
What time is it ten minutes later than 11.55 a.m.?
How many hours are there from 10.0 a.m. to 1.0 p.m.?
5 hr. 20 min. − 2 hr. 30 min.
What time is it half an hour earlier than 12.20 a.m.?

(iv) *Mixed Problem Test based on Weight*

i An empty can weighs 1½ lb. When full of water it weighs 6½ lb. How many gallons of water are there in the can?
ii How many ¼ lb. bags can be filled from 4 lb. of rice?
iii If ½ lb. tea costs 2*s*. 6*d*. what will ¼ lb. cost? What will 1½ lb. cost?
iv How many 2 lb. bags can be filled from ½ cwt. of sugar?
v A can holds 1½ gall. of water. The empty can weighs 2 lb. What will it weigh when full of water?
vi Tom weighs 6 st. 9 lb. Harry is 11 lb. heavier. How heavy is Harry?
vii Eggs are packed 12 in a box. Each egg weighs 2 oz. and the empty box weighs 9 oz. What will the box weigh when full?
viii A cake weighs 1 lb. 8 oz. It is cut into 8 equal parts. What does each part weigh?

## Use of test cards

Often the tests given in class text books are mixed tests or they do not meet the exact requirements of the teacher. Many prefer to compile their own test cards for use on those occasions when they desire to test a particular aspect of the work. These test cards can be grouped into types and numbered within the groups so that the teacher has at his disposal a number of cards on Addition, on Subtraction and so on. Provided that care is taken in compiling the cards so that each card has a definite purpose, then they are better when compiled by the teacher himself, for he knows his pupils and their weaknesses and what he particularly desires to test.

## Oral and mental tests

*Many teachers find it profitable to devote the first five minutes of a lesson to quick mental work.* This prepares the pupil for what is to follow and gets him into the right frame of mind. This method is preferable to using a set period, say once a week, for mental work. Such a period is too long and better results are obtained by this regular daily short period.

Oral tests are particularly useful because they save a great deal of time and they can be varied at will. The speed of working can also be controlled by the teacher and adapted to a particular pupil or to a particular test. If used with oral answering they have the disadvantage that only one pupil actually answers and it is more satisfactory if the answers are written down. This form of testing is generally known as Mental Arithmetic, which is not a particularly suitable title as all Arithmetic is a mental exercise. However, the name is in such common use that provided it is clear that what is meant is Arithmetic without written working then no harm is done by continuing to use the name.

*Mental Tests are particularly useful for testing fundamental knowledge.* A large number of easy questions can be set either by dictation or by writing them on the blackboard and the time for writing down the answers can be easily controlled and varied. In this way it is possible to test, say, all the basic addition bonds in five or ten minutes.

## Other uses of mental tests

There are other uses of these short "mental tests":

i They provide a method for stimulating mental alertness.
ii They provide a quick and convenient method for the revision of work done.
iii They provide a simple method for diagnosing difficulties with the basic facts.
iv They help to diminish the reliance on paper working.
v They help to develop speed in calculation.

These aims can be accomplished only if pupils are encouraged to work quickly without any form of written working. Always insist on a time limit for all mental tests and aim at reducing this time progressively. A good basis for your timing is half a minute per sum for ordinary calculations and less than this for those tests on basic fundamentals such as tables, number bonds and simple multiplication facts. Speed in simple calculation can only be obtained by repeated and regular practice.

## Diagnosing difficulties

The word "diagnose" means to find out what is wrong. Just as a doctor diagnoses the illnesses of his patients, so must a teacher diagnose the mistakes of his pupils. Only when he has discovered what the difficulty is, can he hope to apply the correct remedy. In order to make this task possible and to ensure that the diagnosis of difficulties is carried out without undue loss of time, some system is necessary.

The real purpose of tests designed to diagnose difficulties experienced by pupils is to *analyse* and not to *assess* the pupil's ability. Such tests must be carefully compiled so that they cover all aspects of a particular technique for only then will the results present a complete picture. As an example, consider the diagnostic tests which it will be necessary to set in order to obtain an analysis of each pupil's ability in addition. In order to be comprehensive it will be necessary to test:

i The 100 basic addition facts using the numbers 1 to 9 together with the zero.
ii Addition of numbers over 10 without bridging.
iii Addition of two- and three-figure numbers without bridging.
iv Bridging from units to tens only.
v Bridging from tens to hundreds only.
vi Bridging from hundreds to thousands only.
vii Mixed bridging.
viii More than two-number addition.
ix Horizontal addition.
x Addition using harder numbers.

This would seem to be a formidable list but because it is complete there can be no mistake about the results. The teacher will not have to rely on guesswork or on random results from individual sums. He will have a complete and accurate picture of the difficulties in all forms of addition, and like the doctor he will be able to apply his remedies. In spite of this very long list, these tests are worked quite quickly and in fact the time of working each is important because this is an indication of the degree of difficulty encountered. Thus for the test on the 100 basic addition bonds the time should not exceed five minutes. For the other tests, ten questions each, with a time of 3 minutes for each test should be sufficient. No attempt is made here to give lists of these tests, as the teacher should be able to compile them for himself with the help given above.

Similar tests should be prepared for subtraction, for multiplication, for division and later for fractions and decimals. *In all cases special attention should be given to the tests on the basic number bonds, for on these is built the whole fabric of Arithmetic.*

## Marking Arithmetic

*Right at the start it must be accepted that the work of pupils in Arithmetic must be marked.* Children are interested in knowing whether a sum is right or wrong and they want to know this very soon after they have completed it. Unmarked work leads to a loss of interest and to a repetition of errors which could easily be corrected.

The teacher must face this problem and not dismiss it as an impossibility. It is a very difficult problem, for the teacher has many other duties and responsibilities and the load of marking is heavy.

While there will be some who disagree, *one great help with this marking is to train pupils to use the answer book correctly and honestly*. The usual argument against this is that pupils will be dishonest and copy the answers. It is quite true that some pupils will do this but they will form only a small minority and the teacher, if he is observant, will soon discover them. This method does not absolve the teacher from supervision; it does satisfy the pupil, for he knows whether his sums are right or wrong. Much of the supervision of this marking can be done individually in the classroom while pupils are working. This enables the teacher to deal with an error or a misunderstanding on the spot when the pupil is fresh and when he remembers just what he did.

## Analysing the errors

Merely to mark the sums right or wrong will not help the teacher to discover the cause of the error. Several examples are given to illustrate the common types of error. Many more could be given but these should be sufficient to guide the teacher in his marking:

i $$\begin{array}{r} 343 \\ -127 \\ \hline 226 \\ \hline \end{array}$$

Omitted to allow for the borrowing of a ten (or for adding a ten to the 2 tens)

ii $$\begin{array}{r} 209 \\ \times \quad 8 \\ \hline 1656 \\ \hline \end{array}$$

Error in Tables ($8 \times 9 = 72$ *not* 56)

| | | |
|---|---|---|
| iii | $\begin{array}{r} 62 \\ \times\ 23 \\ \hline 124 \\ 186 \\ \hline 310 \\ \hline \end{array}$ | Error in placing the figures when multiplying by 20. Actual multiplication correct and addition correct. |
| iv | $4\overline{)39}$<br>8 rem. 7 | Remainder larger than divisor—poor knowledge of the tables. |
| v | $\begin{array}{r} 403 \\ -167 \\ \hline 366 \\ \hline \end{array}$ | Difficulty over noughts. After subtracting 7 from 13 correctly he did not know what to do and copied down the 6 tens. |

With problems the correcting is twofold and the teacher must look for poor or cumbersome methods as well as inaccurate calculation. Statements should be short but clear.

### Corrections

Correction of errors made in Arithmetic is first of all the responsibility of the pupil, but it is dangerous to leave such a general statement without some explanation. To set the class the task of correcting the sums they have wrong is really avoiding the difficulty and may well result in a second crop of errors. *Pupils must know where they are wrong, whether they have made a mistake in method or in calculation.* The former might well result from a failure to understand or to interpret the question, while an error in calculation might be due to a number of causes such as a lack of knowledge of the tables or uncertainty about place values. This is one of the reasons why the teacher must inspect and check the work, for it is his duty to set the pupil on the right path and to take action to prevent a recurrence of the error.

Using this main classification many teachers indicate the nature of the error by an M when the error is due to a wrong method and a C when the error is in calculation. Errors in method when they are fairly general in the class can often be explained by blackboard demonstration, but otherwise they should be dealt with individually. Errors in calculation can and should be corrected by the pupil himself.

A great deal can be accomplished if the teacher uses the times when pupils are working exercises to go around his class and discuss errors with individual pupils.

# PART 2: INTRODUCING MATHEMATICS

*"The first mathematicians, then, were practical men, carpenters and builders."*

Sawyer in
*Mathematician's Delight*

During the child's first years at school, teachers willingly devote time and care to expanding his spatial knowledge, that is his knowledge of simple geometry, measurement and even scale-drawing. He is taught to recognise shapes, to make them into patterns, to appreciate or recognise symmetry and to measure. But later on there is a tendency to neglect spatial work for the purely mechanical, numerical arithmetic. This is because the teacher feels he has no time to do more than make sure that his pupils are proficient in all the techniques connected with the "form rules". Yet every effort must be made to include practical work in Geometry, Graphs, Map-making and Algebra. Not only is such a foundation important for the later study of mathematics in a Secondary school, but it provides an excellent and interesting way of illustrating many of the basic processes of arithmetic.

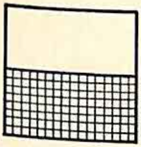
## §7. THE SIMPLE APPROACH TO GEOMETRY

The first steps in the teaching of Geometry come in the years before a child goes to school at all. He builds up his experience of simple shapes: he sees fields of different shapes and sizes; the floor of his house may be a circle or a rectangle, the door is a rectangle and the water-drum a cylinder.

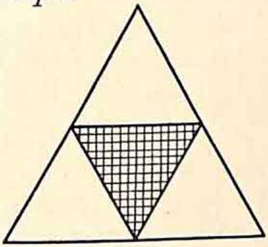
In the classroom the teacher can use this experience and extend it by simple games and by puzzles, using the shapes for making patterns. The pupil can make his own shapes by folding paper or by tracing them from cardboards models. He can compare paper rectangles with objects of similar shape such as a table-top, a door or a window. He can learn about symmetry, that is parts of an object which are the same in size and shape.

Here are some useful exercises:

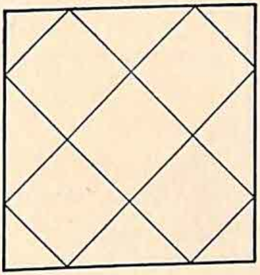
1. *Building shapes*



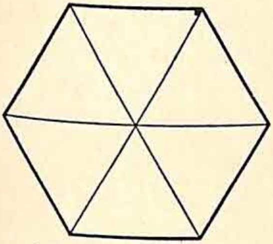
Square from rectangles



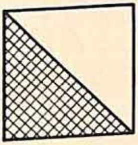
Triangle from triangles



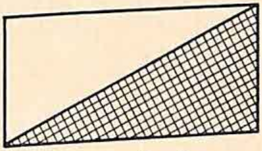
Square from squares and triangles



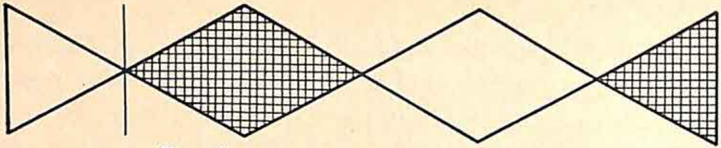
Hexagon from triangles



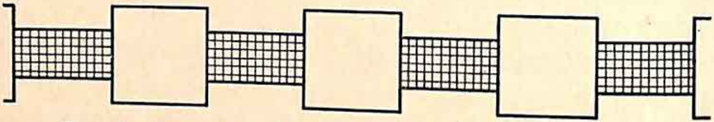
Square from triangles



Rectangle from triangles

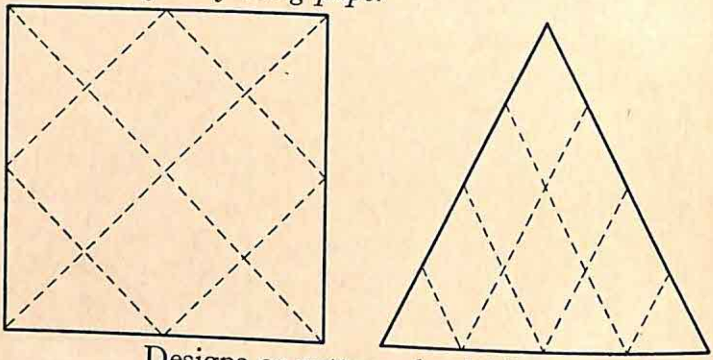


Border pattern from triangles



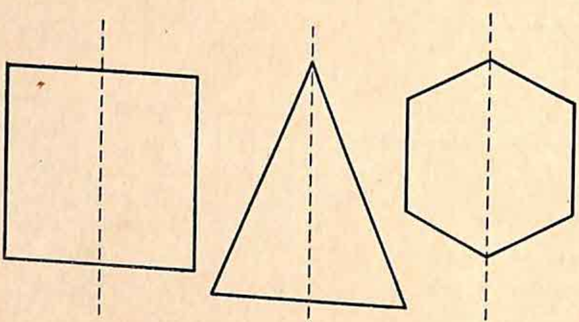
Border pattern from squares and rectangles

2. *Patterns from folding paper*



Designs or patterns by folding

3. *Symmetry*



Symmetry by folding

If these shapes are folded along the dotted line you will see that the two halves are the same size and shape: they are therefore symmetrical.

## Exercises in Practical Geometry

Many useful experiments can be done in the top classes of Primary schools to prepare pupils for the more advanced and abstract Geometry of Secondary schools. Pupils can use written instructions to tackle problems for themselves, and so discover the rules of Geometry for themselves. Of course what they "discover" must not yet be accepted as mathematical proof.

Here are some examples of suitable exercises, some of them illustrated to show the most suitable method:

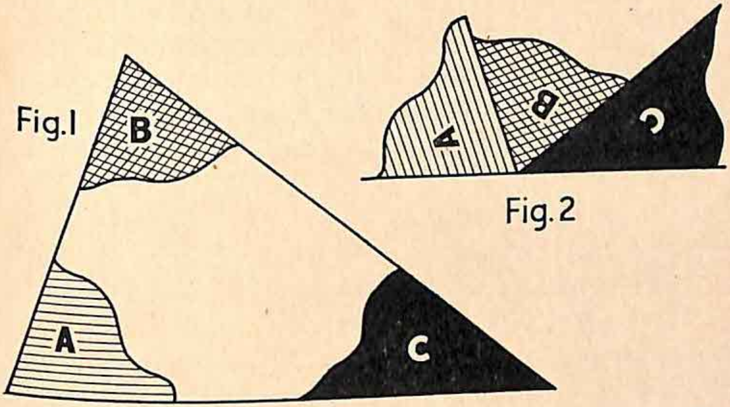


Fig. 1

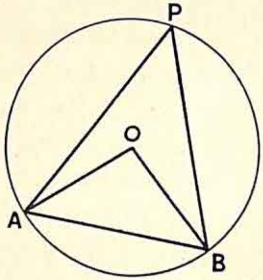
Fig. 2

1. Draw any triangle and mark or colour its angles as shown in fig. 1. Cut out these angles and arrange them as in fig. 2.

Repeat for other triangles of different shapes and sizes.

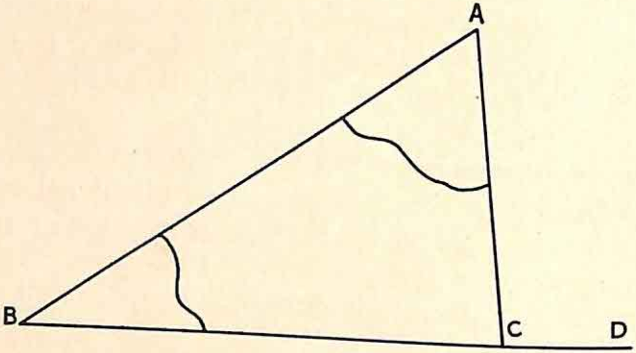
2. Draw a triangle ABC with each side 3 inches in length. By pricking through to another piece of paper cut out an angle equal to one of the angles of the triangle (say angle A). Use this cut-out angle to test the size of the angle B and the angle C.

3. Draw any circle and in it any chord AB. Join OA and OB and also AP and BP where P is any point on the circumference. Cut out the angle P and test the angle AOB, using the angle P as a measure. What can you say about the size of the angle AOB?



4. Draw any right-angled triangle and cut out the two acute angles. Fit them into the right angle. Do they fit exactly?

5. Draw any triangle ABC and continue one side BC to D to form an exterior angle to the triangle. Cut out the angles A and B and fit them into the exterior angle ACD.



Other similar exercises can be constructed by the teacher to cover:

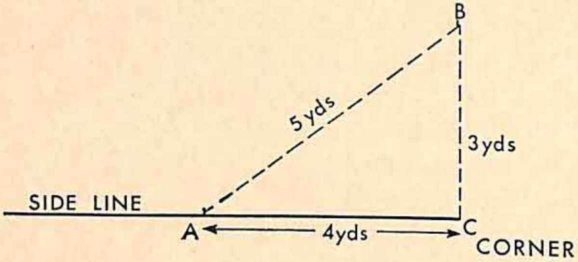
- i The angles associated with parallels.
- ii The angles of a parallelogram.
- iii The sum of the angles of a quadrilateral and polygon.
- iv The angles made by the diagonals of a rhombus.
- v The angles of a hexagon and the shape of its constituent triangles.
- vi The angles at the base of an isosceles triangle.
- vii The sum of each pair of opposite angles of a cyclic quadrilateral.
- viii The angle in a semicircle.
- ix Simple similarity of triangles.
- x Simple constructions for the in-circle and the circum-circle of a triangle.

The value of this type of question in Geometry lies not in the technique, although the ability to draw and construct is involved, but in the valuable training in "doing a job". With this practical experience behind him the pupil will have a far better background for tackling the intricate proofs of Euclid when he reaches the Secondary school. To work these practical exercises successfully the pupil has to interpret and to carry out written instructions. This in itself is valuable training. He has to be accurate and careful in his work and he is introduced to the general principles of making deductions. Here, of course, great care is necessary, for these conclusions are not proofs but rather probabilities.

## Geometry in the Playing Fields

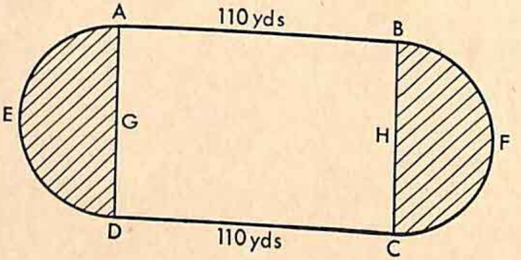
Geometry can also be used to lay out your playing fields or your school garden. Here are two examples:

(1) Making a right angle at the corner of the football ground: a triangle with sides of 3 yds., 4 yds., and 5 yds. will be right angled



From the corner (C) mark off 4 yards along the side-line (CA). Put in pegs at A and C, and tie to them two pieces of string, one 5 yd. long and the other 3 yd. Stretch them and move them until the free ends meet. Where they meet, put in another peg (B). The angle BCA will then be a right angle.

(2) A more complicated example is a running track. A full-size track is $\frac{1}{4}$ mile (440 yd.) round with two straight sides of 110 yd. each and two curves also of 110 yd. each:



AED, BFC are two semicircles. Each has a circumference of 110 yd. and is therefore half of a circle with a circumference of 220 yd. AD and BC are the diameters of the circle.

In a circle the circumference $=2\pi r$

In this case $220=2\pi r$ ($\pi=\frac{22}{7}$)

$\therefore 220\times\frac{7}{22}=2r$ ($2r=$diameter)

$\therefore$ The diameter (AD or BC) is 70 yd.

## §8. SCALE DRAWING, PLANS AND ELEVATIONS

### Scale Drawing

Scale Drawing is useful and important, not only for mathematics, but also for map-work in History, Geography, Rural Science and Agriculture. All pupils must be quite clear that a map or plan drawn to scale is the same shape as the original, but reduced in size in every direction, and that *the same scale must be used for every measurement*.

Give plenty of practice in simple scale drawing—the classroom floor, the school garden, the football ground, etc. At first discuss with the class the most suitable scale to use: later on, leave them to choose their own scale.

Give plenty of exercises such as:

(*a*) If the scale is 1 in. to 1 ft., what lengths will you draw to represent:

2 ft.?
1 yd.?
1 yd., 1 ft.?
5 ft.?
9 in.?
1 ft., 3 in.?
2 ft., 9 in.?
$\frac{3}{4}$ yd.?
90 in.?
60 in.?
1 in.?
2 yd., 1 ft.?

and (*b*) [the same type of question in reverse] Using a scale of $\frac{1}{4}$ in. to 1 ft., what lengths on the ground will be represented on the map by:

$\frac{3}{4}$ in.?
4 in.?
$\frac{1}{2}$ in.?
$1\frac{1}{2}$ in.?
$\frac{1}{8}$ in.?
2·2 in.? etc.

(*c*) Similar questions using other scales.

Scale drawing can also be for maps of journeys: e.g. An aeroplane flies for 100 miles due South and then for 75 miles due East and then flies back to its starting point. Make a scale drawing of its flight, using your own scale and find out by measurement the length of the third part of its flight.

### *Angles and the Compass*

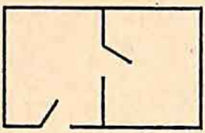
Allied to scale drawing is work on angles and the use of the compass. Often pupils have no very clear idea of what is meant by an angle, and do not realise that it is used to measure the amount of a turn or of a change of direction. Angles are the basis of surveying and of all methods of finding the way by land, sea or air—that is, of navigation.

The study of angles in the Primary school should include:

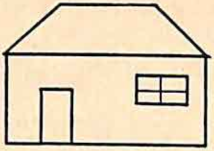
- i The compass, its construction and use.
- ii Bearings and their use in map-making and map-reading.
- iii Simple plans of journeys and flights using the compass or bearings and drawing plans of them to scale.
- iv The measurement of angles of elevation and the measurement of heights by means of the angle of elevation.
- v The measurement and comparison of heights from shadows.
- vi Very simple surveying using the method of offsets (right-angles).

## Plans and Elevations

Most drawings to scale are *plans*, that is to say, pictures of the object from above, but *elevations*—pictures of the object looked at from the side—are also important.



*Plan* of a two-roomed house



*Elevation* of a small house

## Solid Geometry

The study of solid figures such as cubes, prisms and pyramids provides excellent opportunities for practical work. It also allows us to introduce the study of *projection*, that is how to draw solid figures on a flat surface.

But solid geometry, however simple, can only be studied from the solid figures themselves. Large-sized models should be available in every school, and these should be handled and examined carefully. The most satisfactory models are those you can draw on, made of wood or transparent plastic.

Each solid should be studied carefully, with the help of a model and blackboard drawings. The most suitable object to start with is a rectangular prism, because its three views will be of different sizes.

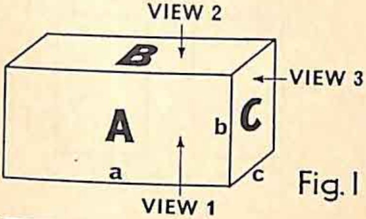


Fig. I

*A rectangular prism* has six faces. In this prism the length is "a" units, the height "b" units and the width "c" units.

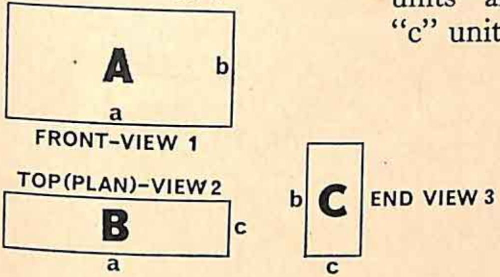
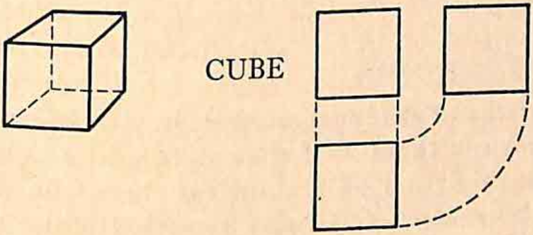


Fig. 2

Look carefully at the six faces. You will find that two are like "A" (with sides "a" and "b"), two are like "B" (with sides "a" and "c") and two are like "C" (with sides "b" and "c").

There is a standard method of drawing the "*projections*" of solids, so that everyone will know what they are. Here are some examples—a *cube*, a *cylinder* and a *pyramid*.

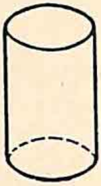


CUBE

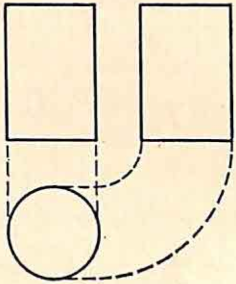
PICTURE

PROJECTION

CYLINDER

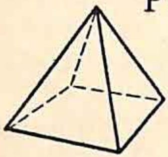


PICTURE

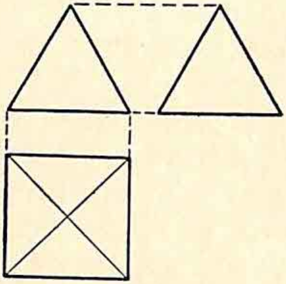


PROJECTION

PYRAMID



PICTURE



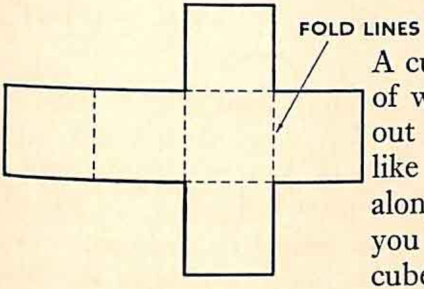
PROJECTION

## *Development of solids*

Another way of studying solids is by *development*, that is by drawing them as if they were opened out. The *development* or *net* of a solid can be easily demonstrated by means of a paper model. If the model is folded correctly it will make the solid shape.

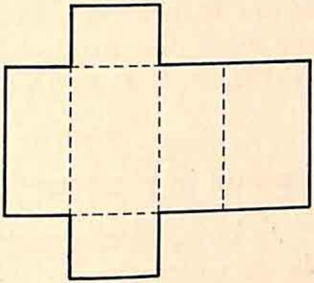
Here are some examples.

(i) *Cube*

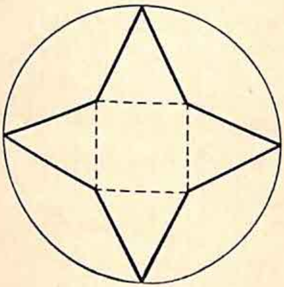


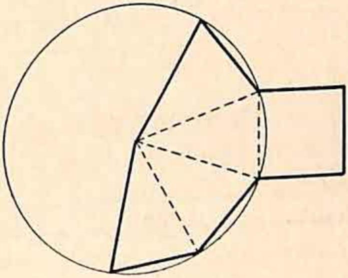
A cube has six faces, each of which is a square. Cut out a model from paper like this picture, fold it along the dotted lines and you will find you have a cube.

(ii) *Rectangular prism*



(iii) *Square Pyramid*





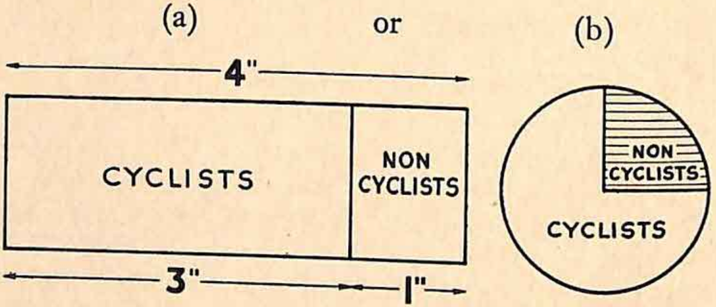
## §9. PICTURE ARITHMETIC AND GRAPHS

A great deal of arithmetic can be expressed in pictures of various kinds, much of it more clearly and in a more interesting way than in figures. If any time is spent on graphs in a Primary school, it is usually confined to the plotting of points or the mechanical drawing of graphs representing formulae in algebra.

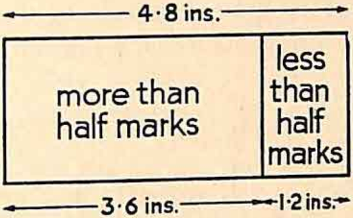
But there is much more to it than this, and, because they are so easy to understand, pictures are being increasingly used in commerce and industry to explain such things as the monthly output of factories.

### i. Simple arithmetic in pictures

(1) In a class of 40 boys, 30 can cycle and 10 cannot cycle. Show this on a picture.



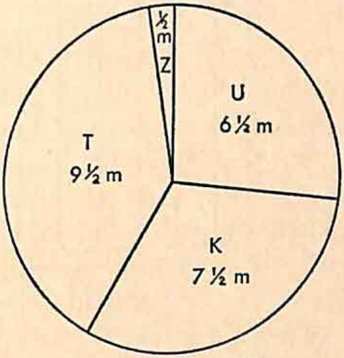
(2) In an arithmetic test, 36 boys out of 48 obtained more than half marks. Show this on a number picture.



(3) The population of the four countries in East Africa is:

Uganda about 6½ million
Kenya about 7½ million
Tanganyika about 9½ million
Zanzibar less than ½ million.

Show this on a "pie" graph.



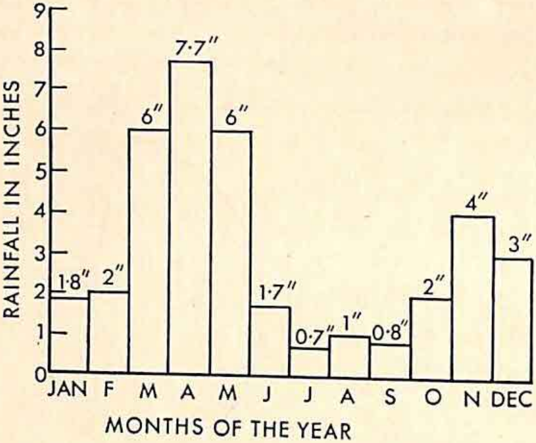
Total 24 million

### ii. Column Graphs

In some cases *column* (or *pillar*) *graphs* are more suitable. These can be used to represent such things as

- *a* Monthly rainfall, hours of sunshine, etc.
- *b* Weekly sales in a shop.
- *c* Annual imports and exports.
- *d* Monthly school expenses; yearly expenditure on (say) education by the central or local government.
- *e* Comparison of heights of mountains, or boys in a class; comparison of lengths of rivers, etc.

Here is an example of a column graph illustrating the rainfall month by month at a school in Africa.



### iii. Straight line graphs

Probably the most familiar straight line graphs are the temperature charts kept for sick people in hospital. Many of the suggestions for column graphs listed in the previous paragraph can equally well be represented by straight line graphs. Other possibilities are:

- *a* Weekly sales in a shop.
- *b* Temperature charts for people and for the weather.
- *c* Attendance figures in the school; weekly marks.
- *d* The weight of a young baby, week by week.

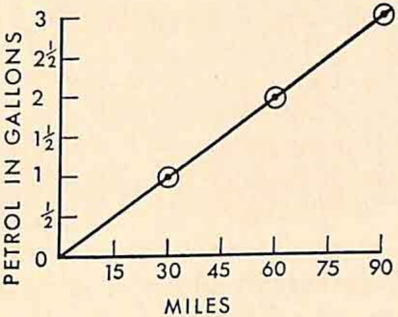
For simple straight line graphs, special graph paper is not necessary. Ordinary squared paper will do perfectly well. These graphs are said to be "*discontinuous*", meaning that there is no connection between one point and the next.

Another important use of straight line graphs is for calculating simple proportion. These graphs are said to be "*continuous*" meaning that there is a definite connection between one point and another.

e.g. A car runs 30 miles on one gallon of petrol. Draw a graph to illustrate this and find out:

*a* How many gallons it uses for 60 miles; 75 miles; 15 miles?

*b* How many miles will it go on 3 gal.; 1½ gal.; ¾ gal.?



This kind of graph can be used for such sums as:

*a* The cost of articles at a given rate.

*b* Perimeter of squares with varying lengths of side.

*c* Length of shadow of a pole during the day.

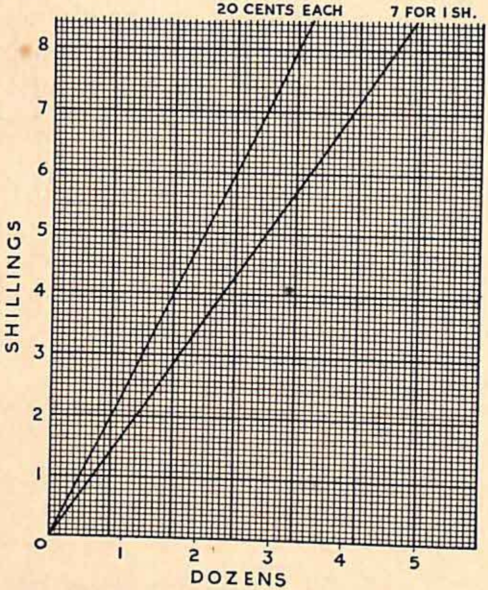
*d* Weight of different volumes of water.

*e* Distance/Time problems.

Here is a more complicated example:

In one shop eggs are 7 for one shilling and in another they are 20 cents each. Show these prices on a graph for any number of eggs up to 4 dozen.

What will be the difference in cost of 2½ doz. eggs at these two prices.



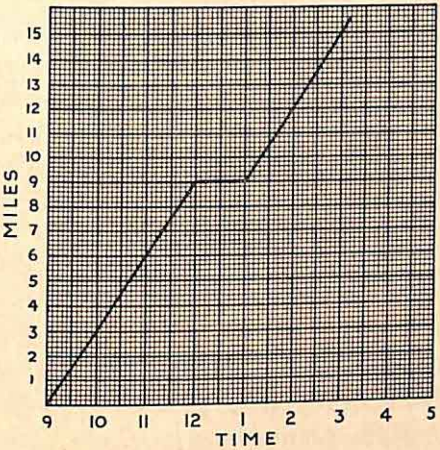
Notes: (*a*) The choice of scale: 10 for the cost axis (since there are 10 ten-cent coins in one shilling) and 12 for the dozens axis.

(*b*) The dearer rate is shown by the steeper graph and (*c*) The difference in cost is shown by the vertical distance between the lines.

Here is an example of a Time/Distance problem, another common kind of sum for which a straight line graph can be used.

A boy walked steadily at 3 m.p.h. from 9 a.m. until noon. He then rests for an hour and then walks on for a further two hours at 3 m.p.h. Draw a graph of this journey and use it to find out:

(*a*) How far he has walked by 11.30 a.m.?
(*b*) How long he has taken to walk 12 miles?
(*c*) Why is his rest period a horizontal line?



## §10. FIRST STEPS IN ALGEBRA

In a Primary school there is no place for much formal Algebra, but it is important to include enough work on both *formulae* and *equations* to make sure that the basic principles are understood and to show that they are neither frightening nor difficult.

Algebra, by the use of letters and symbols, expresses in general terms what Arithmetic, by the use of figures, deals with in particular cases.

### Formulae

An algebraical formula tells us something which is always true. We made use of a simple and well-known formula in the second practical exercise on pp. 78–9 (laying out a running track). We found the diameter and centre of the semicircles at the end of the track by the use of the formula $C=2\pi R$, where $C$ is the circumference of the circle; $R$ is the Radius and $\pi$ (itself a symbol) has a fixed value ($\frac{22}{7}$ or $3\cdot14$).

*In that case:* $C=220$ yd.

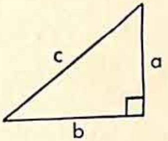
and therefore 220 yd. $=2\pi R$ yd.

and therefore $R$ yd. $=\frac{220}{2\pi}$ yd.

It is important to remember that the formula $C=2\pi R$ is *always true for all circles*, whatever may be the numerical value of $C$ or $R$, and this is the important point about formulae to drive home in the Primary school.

The first practical exercise on p. 78 (setting out a right angle at the corner of a football ground) can also be expressed by means of a formula. In a right-angled triangle $a^2+b^2=c^2$, where $a$ and $b$ are the lengths of the sides containing the right angle and $c$ is the length of the hypotenuse.

In the exercise $a=3$, $b=4$ and $c=5$ units
and so $9+16=25$



but this formula is of course true for all right-angled triangles whatever the length of their sides.

Practice can be given in the use of many other simple formulae, many of them connected with Geometry, such as:

(i) Perimeter of a square: $P=4d$, where $d$ is the length of one side.
(ii) Perimeter of a rectangle: $P=2a+2b$, where $a$ and $b$ are the lengths of the sides.
(iii) Angles of a triangle: $180°=a+b+c$, where $a$, $b$ and $c$ are the sizes of the three angles in degrees.
(iv) Weight of water in pounds: $W=10g$, where $g$ is the number of gallons of water.
(v) Area of a rectangle: $A=ab$, where $a$ and $b$ are the lengths of the sides. In this example the use of $ab$ for $a\times b$ can be explained.
(vi) Area of a square: $A=a^2$, where $a$ is the length of a side. In this example it can be explained that $a^2=a\times a$.

(vii) Area of a circle: $A=\pi R^2$, where $R$ is the radius of the circle.

(viii) Area of a triangle: $A=\frac{1}{2}bh$, where $b$ is the length of the base and $h$, the vertical height.

(ix) The volume of water in a cubical tin: $V=a\times a\times a$ or $V=a^3$, where $a$ is the length of a side of the cube.

Many sums in Arithmetic can be expressed in general terms by formulae, for instance, the cost of $n$ pineapples at $p$ cents each. The formula would be $C=n\times p$ (or $C=np$) cents, or, $C=\frac{np}{100}$ shillings.

It may be of interest to show that a graph can be drawn of this formula for different values of $n$ (or for different values of $p$) and that the graph will be a straight line graph. Similar graphs can be drawn of formulae (i)–(iv) on p. 93, but graphs of sums which contain multiplication such as (v)–(ix) should not be attempted at this stage.

## Equations

In dealing with equations the important point to drive home is that the sign for "equals" (=) means what it says and the figures (or letters or symbols) on one side of it are equal to—or, if you prefer it, balance—those on the other side of it. This may seem obvious, but it needs to be emphasised continually.

Some equations are clearly true:

e.g. $3=3$ *or* $17=10+7$ *or* $\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$.

Some are less clear:

e.g. $\frac{3}{10}+\frac{1}{5}=\frac{1}{2}$ *or* $\frac{19+6}{5}=17-12$.

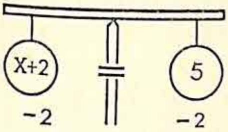
The use of letters and symbols makes no difference, but before introducing this complication, it is worth while spending some time on the comparison with a pair of scales. For this demonstration, you must have a real pair of scales, with real weights of 1 lb., 2 lb., etc., and some packages of known weight such as pounds of tea. You can use packets of sand if you like, but if so, they must really weigh what you say they do.

From these experiments with scales, the following rules can be learnt:

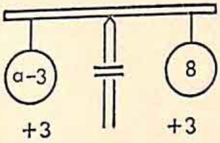
- (*a*) The scales will only balance if what is on one pan is equal in weight to what is on the other.
- (*b*) Only when the two sides balance exactly are you allowed to use the sign "equals" (=).
- (*c*) The two sides will continue to balance as long as you add the same weight to each side or take away the same weight from each side.

When these rules are absolutely clear, and this may take some time, you can proceed to equations which contain letters or symbols. Use very simple examples at first and a diagram of a pair of scales to help.

e.g. $x+2=5$ *or* $a-3=8$



$x=3$



$a=11$

Simple problems can also be dealt with in this way. e.g. "A boy spent 5*s*. and had 8*s*. left in his pocket.

How much had he to start with?"

$x - 5 = 8$

$\therefore\ x = 13.$

This will be the time to point out that you can divide each side of the scales by two or multiply each side by ten or do anything you like, *as long as you do the same to both sides*. And so you will be able to solve such equations as:

(*a*) $2x = 6.$
(*b*) $\frac{1}{2}y = 10.$
(*c*) $8p = 64.$
(*d*) $3x - 6 = 0.$
(*e*) $2x - 5 = 7.$
(*f*) $5g + 4 - 2g = 10.$
(*g*) $5x + 3 = 4x + 6.$
(*h*) $2c + 3 = 15.$

## §11. IN CONCLUSION

> *". . . a child's success depends upon careful planning and grading of the work: the child must proceed gradually, not meeting too many difficulties at the same time, and must be given adequate practice at every stage."*
>
> Downes and Paling in
> *The Teaching of Arithmetic in Tropical Primary Schools*

This short book does not attempt to cover the whole field of the teaching of Arithmetic. The aim has been to introduce the teacher to some of the more important aspects and to encourage him to dig more deeply into other more detailed books on the subject. If the book accomplishes this by stimulating the teacher to give more thought to the planning of his work, by persuading him to study more carefully the opinions and experiences of others, then it will have accomplished its purpose and it will have gone some way towards improving his teaching of the subject in school.

To summarise the main points, those essential factors for successful teaching:

(*a*) First of all must come the necessity for the teacher himself to understand the main principles which should guide his teaching. *He must know what he intends to teach, why it is necessary to teach it and how best the child will learn and understand what has been taught.*

(*b*) *Secondly, the teacher must plan his work carefully* to ensure that progress is steady and that the child proceeds from the simple to the more difficult, one step at a time. Teaching arithmetic can be compared to building a house. First there is a plan which shows not only the various stages in the building but also the completed house. Then the foundations are laid, firmly and solidly. The walls are built slowly and steadily and finally the finishing touches are added. The greater the skill of the workmen, the better the house will be when it is completed.

(*c*) *Thirdly, the teacher must check on the progress of his pupils*. He must discover their difficulties and diagnose their lack of understanding. He must be ready and able to apply a suitable remedy. In this the teacher is like the physician. He must know and understand the symptoms of the illness and he must be ready with the right medicine to produce a speedy and complete recovery.

(*d*) Fourthly, and this is really connected with all the principles given above, the teacher must accept that there is a dual aspect to all Arithmetic teaching. These are closely interwoven and the one is of little value without the other:

i *The necessity for proficiency in techniques.*
ii *The ability to apply the techniques* to solving problems.

Keep these always in mind and you are on the way to becoming a better teacher.

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